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Probabilistic approach to assessing tribotechnical reliability indicators of friction units

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Abstract

The article presents a theoretical and analytical review of the probabilistic approach to assessing the tribotechnical reliability of mechanical systems, in particular friction units. The influence of the random nature of loads and wear on the reliability of machine elements is considered. The feasibility of using distribution functions and probability density functions to describe the wear process is substantiated. Mathematical models are described in detail that allow determining the probability of element failure for given statistical characteristics of the load and permissible wear. Both constant and block load conditions are taken into account. The results of the study can be used in the design and operation of highly reliable tribotechnical systems, as well as for predicting their resource under conditions of operational uncertainty.

Keywords: tribotechnics, wear, probabilistic approach, reliability, failure function, coefficient of variation, block load.

Introduction

Ensuring the reliability of technical systems is one of the main tasks of modern mechanical engineering, in particular in conditions of intensive operation and high loads. Friction units are one of the most vulnerable parts of mechanisms, and their premature failure can cause significant economic losses or even accidents. The traditional approach to reliability assessment often ignores the stochastic features of the operation of tribotechnical elements, which leads to an underestimation of the accuracy of predictions.

Given that the wear process is complex, non-uniform and largely random, there is a need to switch to probabilistic analysis methods. This approach allows us to take into account not only the average values of loads and wear, but also their variations, dispersions and other statistical characteristics. In particular, the use of probability density distribution functions allows us to estimate the probability of failure, construct reliability functions and optimize design parameters.

This article highlights the basic concepts and mathematical tools underlying the probabilistic approach to tribotechnical reliability. This research has practical implications for designers of mechanisms operating under conditions of significant and variable loads.

Purpose and objectives of the study

The aim of the work is to develop a probabilistic model for assessing the reliability of friction units taking into account the random nature of wear and load. To achieve the set goal of the research, the following tasks were solved: to build mathematical models of wear and load distribution; to determine distribution and density functions for key parameters; to propose a methodology for assessing reliability under constant and block loading; to calculate reliability based on statistical characteristics.

Research methods

The work uses methods of mathematical statistics, probability theory, as well as approximate analytical methods (linearization of functions) to estimate the distribution characteristics. Both normal and arbitrary probability density distributions were used to model wear. The analysis was carried out by constructing reliability functions and failure functions for different load modes.

Literature review

Tribological reliability issues in engineering have been widely discussed in the scientific literature. In [1], the main focus is on identifying failure modes and mechanisms. This is especially true for the emerging technology of microelectromechanical systems (MEMS). The focus here is on the mechanism of wear failure and how the methodology was used to create a predictive model. The MEMS device that was emphasized in these studies was a Sandia-developed micromotor with orthogonal electrostatic linear actuators connected to a gear on a hub. The dominant failure mechanism was wear in the sliding/contact zones. A sliding beam-on-post test structure was also used to measure friction coefficients and wear morphology for different surface coatings and environments. The results show that a predictive model of failure time as a function of drive frequency based on wear fits the functional form of the reliability data quite well and demonstrates the benefits of a fundamental understanding of wear. In [2], it is stated that tribological experimental studies have improved significantly in recent years, leading to a significant number of results and, as a result, an increasing number of papers are appearing. The scatter found in the data is often explained by many variables involved in the experiments, namely: the environment (especially humidity), layers of contaminants, differences in test conditions, uncertainty in the evaluation of the results and rarely - the response of the experimental equipment. This work aims to discuss several sources of inaccuracies that lead to the scatter of experimental tribology results. A reliability method is proposed to characterize friction and wear data. Experimental results obtained by unidirectional sliding and microabrasion will be used to support the discussion. In [3], it is stated that system reliability is an extremely important issue, especially in multi-core systems, which tend to have high power density and, consequently, temperature. Existing reliability-based methods are either slow and non-adaptive, or do not use task assignment and scheduling to compensate for the uneven wear state of the core. This paper presents a dynamically activated task assignment and scheduling algorithm based on theoretical results, which clearly optimizes the system lifetime. In the study [4], the effects of the coupling between linear guide wear and vibration of a machine table system are studied based on the infinitesimal method. A nonlinear dynamic model is developed to analyze the wear and vibration failure mechanisms under parameter uncertainty. To assess the dynamic reliability of a machine table system under multiple failure modes, a time-dependent and conditional reliability approach based on the Kriging model with active learning and Monte Carlo simulation is proposed.The approach eliminates the need to recalculate the real values of the limit state function, and the calculation efficiency is significantly increased. The document [5] describes methods for formulating and assessing the reliability of the system in conditions where failure is the result of wear of system parts reaching a critical threshold. A model is proposed related to the stochastic behavior of wear, in which a continuous, right-hand non-decreasing wear process consists of a “continuous” and a “jump” part. Several properties of the model we have proposed are presented. Also, a number of applied problems on wear and reliability are highlighted in the works [6-9]. At the same time, it should be noted a small proportion of works that analyze numerical dependencies for calculating tribotechnical reliability parameters.

Main material.
Wear and tear as a random variable.

The dependence of wear on the friction path even under constant load is random (Fig. 1). At each given friction path, wear at pressure$\sigma$is a random variable. Like any random variable, wear is characterized by a probability distribution $p(u_w)$ the appearance of this wear.

$p(u_w)$ is the density of the wear probability distribution or wear probability uw;

$Q(u_w)$ is the probability distribution function of wear uw, or the probability of wear occurring from zero to the value uw; this function is called the failure function:

$$Q(u_w) = \int_{-\infty}^{\infty} p(u_w)\, du_w, \tag{1}$$

$P(u_w)$ is a reliability function or probability of failure-free operation, i.e. the probability that wear u_w will not be achieved:

$$P(u_w) = 1 - Q(u_w). \tag{2}$$

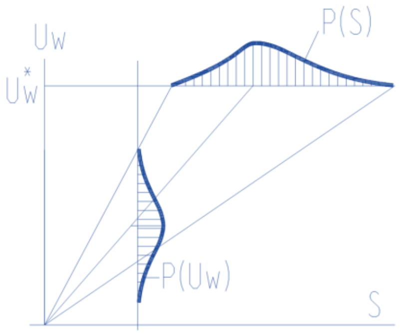

Fig. 1. Random distribution of wear and friction path

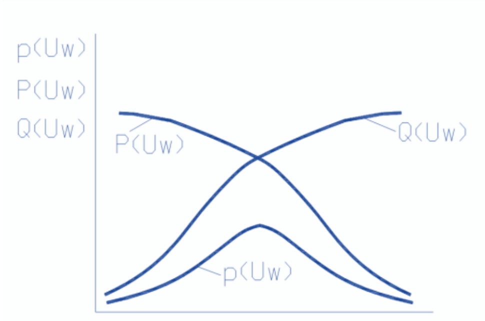

Fig. 2. Reliability functions

The probability distribution density is characterized by:
1) the mean or mathematical expectation

$$\bar{u}_w = m_{u_w} = \frac{1}{N}\sum_{i=1}^{N} u_{wi}; \tag{3}$$

2) dispersion

$$D_{u_w} = \frac{1}{N}\sum\left(u_{wi} - u_w\right)^2; \tag{4}$$

3) standard deviation

$$S_{u_w} = \sqrt{D_{u_w}}, \tag{5}$$

4) coefficient of variation

$$V_{u_w} = \frac{S_{u_w}}{\bar{u}_w}. \tag{6}$$

According to the above formulas, the characteristics of the distributions are determined from experimental data u_w and S_i.

Depending on the wear pressure and the friction path, the random nature of the wear is reflected in the value of the wear coefficient k_w:

$$u_w = k_w \sum_{i=1}^{N} \sigma_i^m \Delta s_i$$

Or in integral form:

$$u_w = k_w \sigma^m s. \tag{7}$$

In this dependence (7), in addition to the random variable k_w, there is, as a rule, the value of the acting pressures σ. Let the system be subjected to a random pressure σ. Random variable σ, as well as k_w, is described by the density distribution $P(\sigma)$ with characteristics: average value $\bar{\sigma}$, pressure dispersion D_σ, the standard deviation S_σ and the coefficient of variation of pressures ν_σ.

It should be emphasized that the random nature of a constantly acting load applies only to the set (set) of friction pairs operating under all other equal conditions, except for the load, the random nature of which is manifested in the random selection of the load at the beginning of the operation of the unit.

The task of constructing the density of the wear distribution for given wear coefficient distributions k_w and pressure σ in formula (7) can practically only be solved approximately. The average wear value $\bar{u}_w$ is calculated by the formula for the average values of the arguments of random $\bar{k}_w$ and $\bar{\sigma}$:

$$\bar{u}_w = \bar{k}_w \bar{\sigma}^m s. \tag{8}$$

To calculate the variance D_{uw} we will use the approximate method of linearization of functions, according to which for the independent variables k_w and σ:

$$D_{u_w} = \left(\frac{\partial \bar{u}_w}{\partial \bar{k}_w}\right)^2 D_{k_w} + \left(\frac{\partial \bar{u}_w}{\partial \bar{\sigma}}\right)^2 D_\sigma \tag{9}$$

or taking into account (7):

$$D_{u_w} = \left(\bar{\sigma}^m s\right)^2 D_{k_w} + \left(\bar{k}_w m \bar{\sigma}^{m-1} s\right)^2 D_\sigma . \tag{10}$$

It is obvious that the constant value s can be placed outside the brackets:

$$D_{u_w} = s^2 \left[\left(\bar{\sigma}^m\right)^2 D_{k_w} + \left(\bar{k}_w m \bar{\sigma}^{m-1}\right)^2 D_\sigma \right]. \tag{11}$$

Reliability assessment from wear under constant load.

The problem is posed as follows: there is a random variable of current wear $u_w(s)$, for example, (7) and the random variable of permissible wear u_w^*, it is necessary to find the probability that the current wear does not exceed the permissible one, that is:

$$P = P\left(u_w < u_w^*\right)$$

or

$$P = P\left[\left(u_w - u_w^*\right) > 0\right]. \tag{12}$$

From probability theory it is known that if the density of the distribution of quantities is given $f_1(u_w)$ and $f_2(u^*{}_w)$, then the distribution density $P(z = u_w - u_w^*)$ is calculated using the integral:

$$P(z) = \int_{-\infty}^{\infty} f_1(u_w) \left[\int_{u_w}^{\infty} f_2\left(u_w^*\right) du_w^* \right] du_w \tag{13}$$

This integral is taken only for certain types of density distributions $f_1(u_w)$ and $f_2(u^*{}_w)$.

In the case where the distribution of effective and limit stresses obeys the normal law for determining $P(s)$, taking the integral (13) is not required in this case.

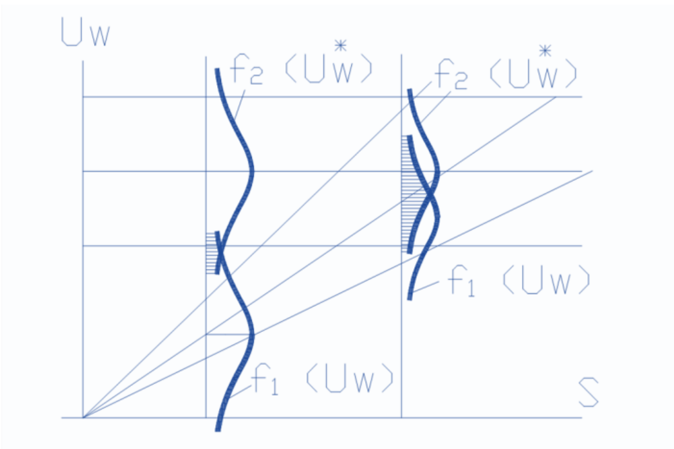

Fig. 3. Normal law of wear distribution

The function $P(s)$ also obeys the normal law:

$$P(s) = \frac{1}{s_z \sqrt{2\pi}} e^{-\frac{(z-\bar{z})^2}{2D_z}}, \tag{14}$$

where:
$z = u_w - u_w^*, D_z = D_{u_w} + D_{u_w^*}, s_z = \sqrt{D_z}, \bar{z} = \bar{u}_w - \bar{u}_w^*.$

The value of the random variable z corresponding to a certain probability $P(s)$ is determined from the expression:

$$z_p = \bar{z} + u_p s_z. \tag{15}$$

where u_p is the quantile corresponding to the probability *P*.

Value $z = u_w - u_w^* = 0$ delimits the regions of negative and positive values of z so that the probability of destruction is determined by the condition:

$$z_p = \bar{z} + u_p s_z = 0,$$

from which the expression for the desired quantile follows:

$$u_p = -\frac{\bar{z}}{s_z} = \frac{\bar{u}_w - \bar{u}_w^*}{\sqrt{D_{u_w} + D_{u_w^*}}}. \tag{16}$$

Entering the depreciation reserve factor:

$$u_p = -\frac{n_w - 1}{\sqrt{n_w^2 v_{u_w^*}^2 + v_{u_w}^2}}; \qquad n_w = \frac{u_w^*}{u_w}, \tag{17}$$

expression (16) is reduced to the form:

$$u_p = -\frac{n_w - 1}{\sqrt{n_w^2 v_{u_w^*}^2 + v_{u_w}^2}}, \tag{18}$$

where $v_{u_w^*}$ and v_{u_w} are the coefficients of variation:

$$v_{u_w^*} = s_{u_w^*} / \bar{u}_w^*, \qquad \mathrm{v}_{\mathrm{u_w}} = s_{u_w} / u_w.$$

According to formula (18), the probability quantile *P* is determined, and then any of the quantities $p(s)$, $P(s)$ and $Q(s)$.

Calculation of wear reliability under random block loading.

Randomness in block loading can manifest itself in:
1) in the random choice of pressure in the stage; 2) in the random choice of interval in the stage; 3) in the random alternation of stages; 4) in the given probability of stages.

Let us consider for example the case where the choice of pressure in a stage is random. Let us assume that in each ith stage:

$$\sigma_i = \bar{\sigma}_i \left(1 + u_p v_\sigma\right) = \bar{\sigma}_i \varepsilon. \tag{19}$$

The size ε, which reflects the random nature of the load, we will assume the same for all stages. Substituting expression (19) into the equation for wear (7), we obtain the statistical expression for wear:

$$u_w = k_w \varepsilon^m \sum \sigma_i^m \Delta s_{i\ddagger} \tag{20}$$

Similarly, substituting expression (19) for the number of blocks before reaching the limit of wear, we obtain:

$$\lambda = \frac{u_w^*}{k_w \varepsilon^m \sum_1^N \bar{\sigma}_i^m \Delta s_{i\ddagger}}. \tag{21}$$

Average number of blocks before reaching wear limit:

$$\bar{\lambda} = \frac{\bar{u}_w^*}{\bar{k}_w \sum_1^N \bar{\sigma}_i^m \Delta s_{i\ddagger}}. \tag{21 a}$$

Average resource of the friction unit:

$$\bar{s} = \lambda \, s_{\ddagger}. \tag{22}$$

It is necessary to find the probability density function of exceeding the wear limit above the current one. Solving this problem in the case of a normal distribution for u_w^* and u_w, is given by expression (18), which can be used to construct the reliability function P(s). In this case, it is necessary to know the wear dispersion Duw, which we find taking into account (20):

$$D_{u_w} = \left(\frac{\partial u_w}{\partial k_w}\right)^2 D_{k_w} + \left(\frac{\partial u_w}{\partial \varepsilon}\right)^2 D_\varepsilon, \tag{23}$$

$$D_{u_w} = \left(\sum \bar{\sigma}_i^m \Delta s_{i\ddagger}\right)^2 D_{k_w} + \left(\bar{k}_w \sum \bar{\sigma}_i^m \Delta s_{i\ddagger}\right)^2 D_\varepsilon. \tag{23 a}$$

Resource distribution taking into account the average value (22):

$$s = \bar{s}\left(1 + u_p v_\lambda\right), \tag{24}$$

where $v_\lambda = s_\lambda / \bar{s}$.

Taking into account (21):

$$s_\lambda^2 = D_\lambda, \qquad \mathrm{D}_\lambda = D_{u_w} + D_{u_w^*} \qquad \mathrm{v}_\lambda = \sqrt{v_{u_w}^2 + v_{u_w^*}^2}, \qquad \mathrm{v}_{\mathrm{u_w}} = \sqrt{D_{u_w}} / \bar{u}_w. \tag{25}$$

The most common is the random load, given in the form of the probability P_i of the load action at each stage. In the case of a continuous load application, this is the pressure distribution density problem $P(\sigma)$. With a discrete load assignment, this is $P_i(\sigma_i)$,so $\sum P_i = 1.$

Conclusions

The probabilistic approach allows for more accurate modeling of the behavior of friction units in real-world conditions. The constructed models take into account the variability of loads and wear, which is crucial for predicting reliability. In particular, when applying the models to block loading, significant differences in reliability functions were found compared to constant loading. The proposed methods can be used to design more reliable machines and mechanisms.

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Диха М.О., Дитинюк В.О., Старий А.Л. Ймовірнісний підхід до оцінки показників триботехнічної надійності вузлів тертя

У статті представлено теоретико-аналітичний огляд ймовірнісного підходу до оцінки триботехнічної надійності механічних систем, зокрема вузлів тертя. Розглядається вплив випадкового характеру навантажень і зношування на надійність елементів машин. Обґрунтовано доцільність використання функцій розподілу та щільності ймовірностей для опису процесу зносу. Детально описано математичні моделі, що дозволяють визначити ймовірність відмови елемента при заданих статистичних характеристиках навантаження й допустимого зносу. Враховано як умови постійного, так і блочного навантаження. Результати дослідження можуть бути використані при конструюванні та експлуатації високонадійних триботехнічних систем, а також для прогнозування їх ресурсу в умовах експлуатаційної невизначеності.

Ключові слова: триботехніка, знос, ймовірнісний підхід, надійність, функція відмов, коефіцієнт варіації, блочне навантаження.