



Theoretical foundations of glow discharge nitriding of internal local recesses on metallic surfaces

M.S. Stechyshyn⁰⁰⁰⁻⁰⁰⁰¹⁻⁵⁷⁸⁰⁻²⁷⁹⁰, O.V. Dykha^{*0000-0003-3020-9625}, M.Ye. Skyba⁰⁰⁰⁹⁻⁰⁰⁰⁶⁻¹⁴⁵¹⁻⁴¹⁸⁶,

D.V. Zdorenko⁰⁰⁰⁹⁻⁰⁰⁰⁴⁻³⁸⁷²⁻³⁷⁹⁷, V.V. Liukhovets⁰⁰⁰⁰⁻⁰⁰⁰²⁻⁶⁹⁷⁸⁻⁷⁸²⁰

Khmelnitskyi national University, Ukraine

**E-mail: tribosensor@gmail.com*

Received: 02 June 2025; Revised 20 June 2025; Accept: 10 July 2025

Abstract

This paper presents a comprehensive analysis of internal local recesses on metallic surfaces during glow discharge nitriding. Various types of recesses and their geometric features are studied to evaluate their impact on electric field concentration, which in turn affects the uniformity of surface modification. A field concentration criterion is introduced to quantify these effects, and an analytical framework is developed to describe the relationship between surface geometry and electric field parameters. The influence of the dimensions of internal recesses on field concentration is examined using numerical simulations. The results provide a deeper understanding of the mechanisms leading to localized variations in current density distribution on nitrided surfaces. The developed analytical tools are applicable for predicting and controlling electric field concentration indicators in glow discharge processes used for the surface modification of metallic components, which is critical for ensuring optimal mechanical and tribological properties.

Keywords: local notches, nitriding, electric field, field concentration coefficient

Introduction

Glow discharge nitriding is a widely used surface modification technique for enhancing the mechanical and tribological properties of metallic components. One of the key factors influencing the effectiveness of this process is the distribution of current density over the treated surface, which directly affects the uniformity of nitrogen diffusion and, consequently, the formation of hard surface layers. The presence of internal local recesses such as grooves, holes, and notches—on functional surfaces leads to significant electric field concentration effects. These localized intensifications of the field can result in uneven heating, local overheating, and the formation of areas with inferior physical and mechanical properties, which compromise the overall performance of nitrided parts.

While the theoretical aspects of field concentration around external surface features have been extensively studied, there is limited understanding of the behavior of internal recesses, especially in the context of complex geometries found in real engineering components. Addressing this gap, this study focuses on developing an analytical foundation for describing and predicting electric field behavior in the vicinity of internal local recesses during glow discharge nitriding. A mathematical model is formulated to capture the interplay between surface geometry and field distribution, allowing the identification of critical parameters that influence current density concentration. The findings provide valuable insights for optimizing nitriding technologies for parts with complex geometries, ensuring a homogeneous and high-quality surface layer.

Literature review

The theoretical foundations of the interaction of the electric field with external local surface recesses are considered in [1-3]. Regarding issues related to the theory of internal local surface recesses and modeling of nitriding of complex surfaces, general information about them and the basic principles of solving the problem are given [4, 5-10].



The objective of the study [5] is to analyze the appearance, mechanisms, and processes responsible for the formation of the second plateau in the nitrogen penetration depth profile during thermal annealing after nitriding. The main hypothesis suggests that the second plateau arises due to the formation of nitrides, which is supported by X-ray diffraction spectra showing the presence of chromium nitride after annealing. A new mathematical model has been developed for nitriding, incorporating non-Fickian diffusion mechanisms and post-nitriding annealing. Simulations performed using the proposed model demonstrate that under certain conditions, a second plateau emerges, confirming the model's ability to reproduce nitrogen depth profiles with two plateaus. The paper provides a detailed analysis of these conditions and quantitatively investigates the influence of various parameters (diffusion coefficient, internal lattice stresses, processing time, etc.). Mathematical modeling of mass transport during gas nitriding was carried out in [6] through numerical calculations in the present study. The diffusion coefficient of nitrogen in 38CrMoAl steel and the mass transfer coefficient in the interfacial reaction were determined. Due to the significant difference between the nitrogen activity at the workpiece surface and the gas phase activity during the nitriding process, it is challenging to control the nitrogen potential and maintain a balanced nitrogen activity. To address this issue, a dynamic control of the nitrogen potential using a computer-based system is proposed. Under high nitriding rates, the computer-controlled technology applied in practical production demonstrates excellent reproducibility and enables precise regulation of the nitrogen potential, thereby reducing the brittleness of the nitrided layer. In [7], the relationships between process parameters and layer structure were established to support the development of software for a process control system aimed at achieving a complex layer structure and optimizing the kinetics of its formation and growth. A concept for a gas nitriding process control system was proposed, based on the synergistic integration of an experimental-theoretical process model and data from a magnetic sensor that detects the nucleation and growth of the layer. The article [8] focuses on the development of an advanced probabilistic model for the gas nitriding process of steel, utilizing a cellular automata framework. This model integrates two interconnected cellular automata that represent the two primary phenomena occurring during gas nitriding: the interstitial diffusion of nitrogen in iron and the structural-phase transformations within the nitride material. By adjusting parameter values, this approach enables the model to be adapted for describing various scenarios of the nitriding process. Low-temperature nitriding of steel or iron leads to the formation of expanded austenite, a solid solution with a high concentration of nitrogen interstitially dissolved in the fcc lattice. The nitrogen depth profiles in this phase typically exhibit plateau-like shapes, which cannot be described by standard diffusion models for semi-infinite solids, requiring a new approach. In [9], a model of interdiffusion in a viscoelastic solid, based on the Maxwell framework, is proposed. It combines mass conservation, Vegard's rule, and the Darken bi-velocity method. In the one-dimensional case, the original differential-algebraic system is reduced to a simpler differential system, facilitating analytical and numerical analysis. The resulting nonlinear coupled problem is solved numerically, and a series of simulations demonstrates its applicability. In [10], the coupled processes of burnishing and nitriding are analyzed. A mathematical model is developed to describe the evolution of stress and deformation states in the material under varying technological conditions. The proposed diffusion model for the nitriding process incorporates specific stages occurring in the surface layer of a complex material structure. Numerical simulations are presented, investigating how the initial deformation state influences nitrogen diffusion and the surface layer, as well as how diffusion processes affect the final residual stress distribution in the surface layer.

Despite significant progress in the study of glow discharge nitriding processes, several critical issues remain unresolved. Most existing studies focus on external surface features such as edges and protrusions. The influence of internal recesses (grooves, holes, slots) on electric field concentration and current density distribution remains insufficiently explored, especially for complex geometries typical of functional surfaces in engineering components. Theoretical approaches describing the interaction of electric fields with simple geometries cannot be directly applied to parts with intricate internal features. There is a need for analytical frameworks capable of predicting field concentration effects in such regions to avoid local overheating and degradation of mechanical properties. Although several mathematical models have been developed for nitriding processes, their validation under practical technological conditions, particularly for surfaces with complex internal recesses, is limited. This gap restricts their application in optimizing real-world processes.

The relevance of this research lies in addressing these gaps by developing an analytical and numerical framework to describe electric field concentration phenomena in internal local recesses during glow discharge nitriding. Solving this problem is essential for ensuring the uniformity and quality of surface modification in metallic components with complex geometries. The outcomes of this study can significantly contribute to advancing nitriding technologies and improving the functional performance and durability of engineering parts in various industries.

Objectives of the Study

The objectives of this study are as follows:

1. To develop an analytical model describing the interaction between the electric field and internal local recesses on metallic surfaces during glow discharge nitriding. The model should account for the geometrical features of surface recesses and enable the determination of electric field concentration in areas with complex configurations.

2. To conduct numerical simulations and analyze the impact of the size and shape of internal local recesses on the distribution of current density and electric field concentration, aiming to optimize nitriding process parameters and ensure uniform surface modification.

Purpose of work

Development of analytical foundations of the interaction of the electric field with internal local surface recesses.

Research results and discussion

The most general variant was chosen as a model – an internal local notch of a wedge-shaped shape (Fig. 1). The dimensions of the wedge-shaped groove, especially the width at the entrance, are comparable to the width of the cathode drop region (CDR). Depending on this ratio, the CDR inside the groove can partially or even completely (in the direction of the groove depth) overlap each other. Let us introduce the concepts of zones of complete (A), partial (B) overlap and the zone of ordinary discharge (C). The nature of the overlap, as will be shown below, plays a significant role in determining the trajectory of the electrons that have flown from the surface, since in zones A and B they are under the force influence of two fields from the two walls of the groove at the same time. Moving in the gap between these walls under the influence of a spatial system of forces, electrons can cross the line of symmetry of the groove, i.e. the trajectory of their movement corresponds to a certain extent to oscillatory motion, which significantly increases the length of the path that the electron will travel compared to the movement along the normal to the surface in the CDR of a conventional discharge. Accordingly, the probability of ionization processes increases, which in turn leads to a local increase in the discharge current.

It is obvious that the ratio of the lengths of the overlap zones varies significantly depending on the comparison of the groove width with the CDR width δ . The critical limit is the ratio of the equality of half the groove width b and the CDR width, at which the entire internal space of the groove is a zone of complete overlap.

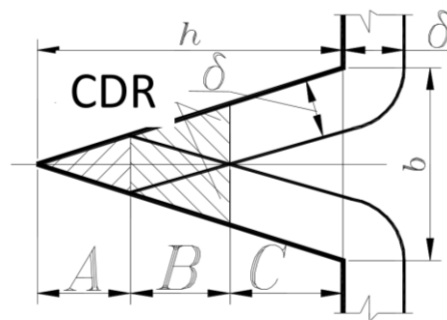


Fig. 1. Scheme of internal local surface recess (A - zone of complete discharge overlap, B - partial, C - ordinary discharge)

When moving to a rectangular groove or hole, the features of the CDR overlap should be taken into account, as well as the known fact of a discharge with a hollow cathode. That is, the field distribution to a depth of no more than double the width of the groove or two diameters of the hole has been experimentally established.

The design scheme of the internal local surface recess in the form of a wedge-shaped groove is shown in Fig. 2.

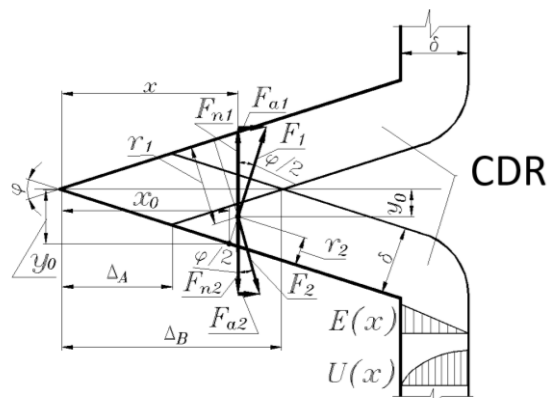


Fig. 2. Calculation diagram of the internal wedge-shaped groove

Groove size ratio (Fig. 1):

$$b/2h = \operatorname{tg} \phi/2 \text{ from where } \phi = 2 \operatorname{arctg} b/2h.$$

Changing the width of the upper edge of the groove: $y = x \cdot \operatorname{tg} \phi/2$,

And the lower one: $y = x \cdot (-\operatorname{tg} \phi/2) = x \cdot \operatorname{tg}(-\phi/2)$.

In zones of complete or partial overlap of fields, an electron that has flown from any point on the face (for example, from the bottom) and whose initial coordinates are x_0, y_0 , can be simultaneously affected by three electric fields: transverse from two faces E1 and E2 and longitudinal E_a . Taking into account the adopted linear law of distribution of transverse field intensity $E(x)$ and the parabolic law of change of voltage drop in CDR – $U(x)$, the field intensity values are:

$$E_1 = \frac{2U_K}{\delta} \left(1 - \frac{r_1}{\delta} \right), \quad (1)$$

$$E_2 = \frac{2U_K}{\delta} \left(1 - \frac{r_2}{\delta} \right), \quad (2)$$

$$E_a = \frac{2U_K}{\Delta_B} \left(1 - \frac{x}{\Delta_B} \right). \quad (3)$$

where U_K is the cathode voltage drop; δ is the CDR width; r_1, r_2 are the current distances from the electron to the groove faces; Δ_B is the distance from the groove top to the boundary of the partial overlap region:

$$\Delta_B = \frac{\delta}{\sin(\operatorname{arc} \operatorname{tg}(b/2h))}.$$

Current distances of the electron to the faces:

$$r_1 = \frac{0,5bx + hy}{\sqrt{\frac{b^2}{4} + h^2}}, \quad r_2 = \frac{-0,5bx + hy}{\sqrt{\frac{b^2}{4} + h^2}}.$$

Longitudinal force acting on an electron:

$$F_a = q_e E_a,$$

where q_e is the charge of the electron.

The resulting force acting on the electron in the transverse direction is:

$$F_n = F_{n1} - F_{n2} = F_1 \cos \frac{\phi}{2} - F_2 \cos \frac{\phi}{2} = q_e \cdot \cos \frac{\phi}{2} \left(\frac{2U_K}{\delta} \left(\frac{r_2}{\delta} - \frac{r_1}{\delta} \right) \right). \quad (4)$$

Resultant force in the longitudinal direction:

$$F_a = F_{a1} + F_{a2} + q_e E_a = F_{n1} \sin \frac{\phi}{2} + F_{n2} \sin \frac{\phi}{2} + q_e \frac{2U_K}{\Delta_B} \left(1 - \frac{x}{\Delta_B} \right). \quad (5)$$

The sum of the projections of all forces acting in the horizontal direction:

$$F_a = F_{a1} + F_{a2} + q_e E_a = F_{n1} \sin \frac{\phi}{2} + F_{n2} \sin \frac{\phi}{2} + q_e \frac{2U_K}{\Delta_B} \left(1 - \frac{x}{\Delta_B} \right). \quad (6)$$

The sum of the projections of all forces onto the horizontal axis:

$$\sum X_i = m_e \frac{d^2 x}{dt^2} = F_a. \quad (7)$$

The sum of the projections of all forces onto the vertical axis:

$$\sum Y_i = m_e \frac{d^2 y}{dt^2} = F_n. \quad (8)$$

Integrating twice, we obtain for the horizontal axis:

$$\frac{d^2 x}{dt^2} = \frac{F_a}{m_e}; \quad \frac{dx}{dt} = \frac{F_a}{m_e} t + C_1; \quad x = \frac{F_a}{2m_e} t^2 + C_1 t + C_2. \quad (9)$$

For the vertical axis:

$$\frac{d^2 y}{dt^2} = \frac{F_n}{m_e}; \quad \frac{dy}{dt} = \frac{F_n}{m_e} t + C_3; \quad y = \frac{F_n}{2m_e} t^2 + C_3 t + C_4. \quad (10)$$

The integration constants are determined by taking into account that at $t=0$,

$$\frac{dx}{dt} = 0; \quad x = x_0; \quad C_2 = x_0; \quad C_1 = 0; \quad x = \frac{F_a t^2}{2m_e} + x_0, \quad (11)$$

$$\frac{dy}{dt} = 0; \quad y = y_0; \quad C_4 = y_0; \quad C_3 = 0; \quad y = \frac{F_n t^2}{2m_e} + y_0. \quad (12)$$

The field concentration coefficient is defined as the ratio of the total length of electron trajectories within the overlap zone to the total length of electron trajectories on a flat section of the face line, the length of which is $\frac{\delta}{\tan(\phi/2)}$.

Considering that

$$t^2 = \frac{1}{\frac{F_a}{2m_e}} (x - x_0) = \frac{1}{\frac{F_n}{2m_e}} (y - y_0), \quad (13)$$

then

$$\frac{2m_e}{F_a} (x - x_0) = \frac{2m_e}{F_n} (y - y_0); \quad \frac{(x - x_0)}{F_a} = \frac{(y - y_0)}{F_n}; \quad \frac{y}{F_n} = \frac{(x - x_0)}{F_a} + \frac{y_0}{F_n},$$

where

$$y(x) = \frac{F_n}{F_a} (x - x_0) + y_0.$$

Thus, the field concentration coefficient for internal local recesses in the form of a wedge-shaped groove:

$$K_E = \frac{1}{\delta} \int_0^{\delta/\tan(\phi/2)} \left(\int_{x_0}^{\Lambda_B} \sqrt{1 + \left(\frac{d(y(x))}{dx} \right)^2} dx \right) dX. \quad (14)$$

Based on the obtained dependence, a computer program was created to determine the electric field concentration coefficients for local recesses of parts to prevent cases of arc discharge.

Conclusions

1. The study provides a detailed analysis of the influence of internal local recesses on the electric field concentration during glow discharge nitriding of metallic surfaces. The developed analytical model describes the

relationship between the geometry of internal recesses and the parameters of the electric field, enabling the prediction of field concentration effects in zones with complex configurations.

2. Numerical simulations performed within the proposed framework demonstrate how the dimensions and shapes of internal recesses affect current density distribution and electric field intensification. The results confirm that sharp-edged and narrow recesses significantly increase field concentration, potentially leading to localized overheating and non-uniform nitriding layers.

3. The introduced field concentration criterion and the analytical apparatus allow precise calculation of electric field concentration indicators, supporting the optimization of nitriding process parameters for components with complex geometries. This approach can help mitigate adverse effects such as local surface tempering and ensure homogeneous surface modification.

4. The developed methodology can be applied in industrial settings to predict critical areas susceptible to arc discharge and to design technological solutions that minimize these risks. By integrating the proposed analytical tools into process control systems, it is possible to enhance the reproducibility and quality of glow discharge nitriding for metallic parts with internal recesses.

References

1. Pastukh IM External local notches of metal surfaces and their influence on the parameters of the modification regime. Bulletin of TUP: Khmelnytskyi, 2001, No. 3, Part 1, Technical Sciences. P. 43-47.
2. Pastukh IM, Lukyaniuk MV, Kurskaya VO Initial provisions for determining electrical characteristics during nitriding in a glow discharge with non-stationary power supply. Bulletin of Khmelnytskyi National University. Technical Sciences. 2012. No. 1. P. 7-10.
3. Pastukh IM Theory and practice of hydrogen-free nitriding in a glow discharge. Kharkov: National Scientific Center "Kharkov Physical and Technical Institute", 2006. 364 p.
4. Methodology and results of the study of physical, mechanical and tribological characteristics of nitrided inner surfaces of long holes / M. Stechyshyn, O. Dykha, N. Stechyshyna, and D. Zdorenko // Problems of Tribology. -2024. – 29, No. 2/112. – P. 23–30.
5. Galdikas, A., Andriūnas, P., Czerwiec, T., Marcos, G., & Moskaliovienė, T. (2025). Modeling of Plasma Nitriding and Thermal Annealing Processes of Austenitic Stainless Steel. *Physica B: Condensed Matter*, 417487.
6. Hu M.-J., Pan J.-S., Li Y.-J., & Ruan, D. (2000). Mathematical modeling and computer simulation of nitriding. *Materials science and technology*, 16(5), 547-550. <https://doi.org/10.1179/026708300101508054>
7. Ratajski, J., & Suszko, T. (2008). Modeling of the nitriding process. *Journal of materials processing technology*, 195(1-3), 212-217. <https://doi.org/10.1016/j.jmatprotec.2007.04.133>
8. Ratajski, JZ, & Mydlowska, KA (2024). Mathematical modeling of gas nitriding process of steel using cellular automata. *Journal of Achievements of Materials and Manufacturing Engineering*, 125(2). <https://orcid.org/0000-0001-8552-8266>
9. Bożek, B., Sapa, L., Tkacz-Śmiech, K., Danielewski, M., & Rybak, J. (2022). A Mathematical Model and Simulations of Low Temperature Nitriding. *CMES-Computer Modeling in Engineering & Sciences*, 130(2).
10. Skalski, K., Wróblewski, G., & Piekarski, R. (2025). Modeling of residual stresses in surface layer treated by burnishing and nitriding. *WIT Transactions on Engineering Sciences*, 25.

Стечишин М.С., Диха О.В., Скиба М.Є., Здоренко Д. В., Люховець В.В. Теоретичні основи азотування у тліючому розряді внутрішніх локальних виїмок металевих поверхонь

Розглянуто типи локальних заглиблень поверхні, проаналізовано їхній вплив на концентрацію електричного поля та його зміну як чинник нерівномірності результатів модифікації. Запроваджено критерій концентрації поля, розв'язано аналітичну задачу щодо взаємозв'язків між геометрією поверхні та параметрами електричного поля. Досліджено вплив розмірів внутрішніх локальних заглиблень на концентрацію електричного поля. Отримані результати можуть бути використані для точного визначення розподілу щільності струму по азотованій поверхні. Розроблений аналітичний апарат може бути застосований для розрахунку показників концентрації електричного поля у тліючому розряді, що використовується для модифікації поверхонь металевих деталей.

Ключові слова: локальні виїмки, азотування, електричне поле, коефіцієнт концентрації поля