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Analytical study of a hydraulic drive model for a municipal waste container overturning mechanism in a garbage truck considering the wear of friction pairs

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Abstract

The article is dedicated to the analytical study of the improved mathematical model of the hydraulic drive of the mechanism for overturning a container with municipal solid waste into a garbage truck, taking into account the wear of friction pairs. As a result of the analysis of the performed numerical studies of the nonlinear improved mathematical model of the hydraulic drive of the mechanism for overturning a container with municipal solid waste into a garbage truck, taking into account the wear of friction pairs, a linearized version of this model was developed in the form of a system of ordinary linear differential equations of the 2nd order. To carry out design calculations of new garbage truck designs, approximate analytical dependencies were obtained for the pressure in the hydraulic cylinder pressure line, the angular velocity and the angle of container overturning on time were obtained based on the proposed linearized mathematical model of the hydraulic drive of the container overturning mechanism in the technological operation of loading municipal solid waste into the garbage truck during the 1st phase – overturning the container to the equilibrium position, taking into account the wear of friction pairs. A regression equation was obtained, which allows to approximately determine the duration of the 1st phase – the rotation of the container to the equilibrium position during its overturning in the technological operation of loading municipal solid waste into a garbage truck, taking into account the wear of friction pairs, which can be used during design calculations of new garbage truck designs taking into account the wear of working bodies without the need to study the nonlinear mathematical model of the drive, as well as during optimization of the main parameters of the hydraulic drive. It was established that the analytical study of the linearized improved mathematical model of the hydraulic drive of the mechanism for loading municipal solid waste into a garbage truck, taking into account the wear of friction pairs during the 2nd phase – pouring waste from the container into the body of the garbage truck requires further research.

Keywords: linearized mathematical model, consideration of wear, Laplace transformation, hydraulic drive, wear, friction units, container overturning mechanism, garbage truck, solid waste.

Introduction

Among the current tasks facing modern municipal engineering in Ukraine, the improvement of mobile manipulator-type equipment, which includes garbage trucks, is particularly important [1]. In this field, ensuring increased wear resistance, reliability, and durability of machine parts is so important, since these characteristics directly affect the efficiency of equipment operation, reduce repair and maintenance costs, and increase the service life of equipment under conditions of intensive use [2, 3]. In Ukraine, the process of collecting and transporting municipal solid waste (MSW) to places of further processing or disposal is mainly carried out using body garbage trucks. The main working body of such machines is loading mechanisms designed in the form of hydraulically driven manipulators [4]. Currently, there are about 3700 garbage trucks in operation, which not only transport waste but also compact it. This significantly reduces transportation costs and the area of landfills required for waste disposal, which is important both economically and environmentally. During the technological operation of MSW loading into the body of a garbage truck, friction components, in particular hinge joints and hydraulic cylinders of the manipulator, are subjected to significant loads. The intensive wear of these elements is due to a number of factors: the large weight of waste containers, which can reach 500 kg, the need for mechanisms to operate in reverse mode with reciprocating movements, and the high number of operating cycles performed during a single trip. An additional complicating factor is the operating conditions, which are characterized by significant fluctuations in relative humidity and temperature, as well as an increased level of dust in the environment. The combination of these factors leads to accelerated wear of working elements, which negatively affects the reliability and durability of garbage trucks. Deterioration of the operational characteristics of parts or insufficient lubrication leads to increased friction forces in the manipulator's hinge joints. This, in turn, causes an increase in the level of vibration in the system, which negatively affects its dynamic stability and reduces the mechanism's ability to withstand high loads under conditions of reversible friction. Accelerated wear of friction components not only reduces the efficiency of the garbage truck manipulator, but also creates potential risks for the safe operation of the equipment. This situation can lead to emergency operating modes that threaten the health of operators and can also have a negative impact on the environment in the event of uncontrolled spillage or leakage of solid waste. According to the Resolution No. 265 of the Cabinet of Ministers of Ukraine [5], one of the key areas of development of public utilities is the introduction of modern, highly efficient garbage trucks. Such vehicles are considered a central link in the system of technical means for the collection, transportation, and primary processing of solid waste. The use of the latest models of equipment not only optimizes logistics processes and reduces operating costs, but also contributes to the comprehensive solution of a number of pressing environmental problems related to waste management. In addition, the modernization of the garbage truck fleet ensures an increase in the overall reliability and efficiency of the country's public utilities, which is of strategic importance for the sustainable development of settlements. The planning of the renewal, maintenance, and repair of garbage trucks is facilitated by the determination of approximate analytical dependencies of the main power and kinematic characteristics of the hydraulic drive of the mechanism for overturning containers with solid household waste into a garbage truck, taking into account the wear of friction pair.

Analysis of recent research and publications

In the article [6], a structural analysis of the wear parts of a garbage truck is presented. The authors of the paper [7] proposed a method for diagnosing faults associated with the wear of the hydraulic cylinder seal and internal leaks of the working fluid, based on the fusion of energy characteristics. In the paper [8], a scheme of a robotic manipulator was developed, its 3D geometric model was created using SOLIDWORKS, and the motion analysis was performed. During the design of structural optimization, key parameters such as stress and deformation of the robotic manipulator were analyzed in detail, and objective functions and constraints were established for optimizing and improving its structure. This allows to reduce the failure rate of the manipulator during long-term high-intensity operation and better achieve joint operation with other systems of the garbage truck to increase the overall efficiency of MSW collection and transportation.

In the study [9], a method for optimizing the operation of a robotic cell is presented, which involves changing the position of the robot manipulator within the working area for programs with a fixed trajectory of the endpoint. The main goal was to reduce the total wear of the manipulator joints and prevent their uneven loading, when individual joints are subjected to greater stress compared to others. The assessment of wear was carried out by approximating the integral of the mechanical work of each joint along the entire trajectory, which is determined through angular velocities and applied torques. The proposed approach is based on the usage of dynamic modeling, which allows determining the torques and rotation speeds of the joints in different positions of the robot. The results of the study showed that the optimal location of the manipulator base allows reducing the overall level of wear of its joints by 22-53%, depending on the configuration of the trajectory of movement.

In the article [10], a mathematical model was developed that allows determining the optimal geometric parameters of the manipulator's structural elements, taking into account the maximum boom reach, load capacity and a number of other kinematic characteristics of the machine. Such a model is an important tool for design engineers, since it provides the possibility of rational selection of dimensional parameters of structural elements in order to increase the efficiency of the manipulator and ensure its reliability during operation. At the same time, special attention was paid to the peculiarities of the operation of articulated joints that operate in a cyclic mode, which is typical for manipulator-type machines. It was established that under such conditions, the formation of a complete hydrodynamic friction mode is impossible, since the lubrication process occurs mainly in the semi-dry and boundary friction modes. This leads to increased requirements for the properties of the materials of the parts, the quality of the surface finish and the efficiency of the lubrication system, since it is these factors that determine the wear resistance and durability of articulated joints in real operating conditions. Unlike the steady-state mode of hydrodynamic friction, the operation of sliding bearings in conditions of semi-dry or boundary friction is accompanied by more intense wear of the friction surfaces. This leads to a gradual loss of kinematic accuracy, the appearance of additional dynamic loads, impact loads and vibrations, which, in turn, cause the development of fretting corrosion and premature destruction of parts. To reduce the friction force, it is proposed to use special coatings of the conjugate elements of the manipulator hinges, in particular lead, phosphate and indium. It has been proven that the intensity of contact wear can be significantly reduced by using lubricants based on oils and fats, as well as greases, which at a temperature of 25 °C acquire a thick, ointment-like consistency. In addition, the feasibility of using phosphate and anodic metal coatings has been determined, which contribute to better retention of lubricants on friction surfaces, increasing their efficiency and durability of components.

In the work [11], a detailed analysis of the main types of wear of hinged joints used in the structures of forest manipulators was carried out. The obtained research results made it possible to determine promising directions for increasing their wear resistance, which can be used by design engineers in order to extend the working life of the mentioned units depending on the specific operating conditions and requirements for the equipment. Particular attention is paid to the fact that manipulator machines mostly operate in difficult climatic conditions with sharp fluctuations in ambient temperature. Such factors significantly affect the stability of the properties of lubricants and the operational characteristics of the structural materials of the hinges, which ultimately causes accelerated wear and a decrease in the durability of the machines. Under conditions of reduced temperatures, the properties of the materials of friction pairs undergo significant changes. In particular, an increase in their fragility, a decrease in the yield point, and an increase in the rigidity of the working surfaces are observed. This leads to a complication of the processes of movement and annihilation of dislocations in the crystal lattice of the material, which is accompanied by the appearance of exoelectronic emission and a significant acceleration of the wear processes of friction surfaces. An additional negative factor is a change in the characteristics of lubricants: at low temperatures they can lose their fluidity, turn into a solid state or significantly increase their viscosity. As a result, the lubricant loses its ability to form a reliable film between the working surfaces, which sharply reduces its protective and antifriction properties and leads to an intensification of wear processes. In the summer, when the ambient temperature reaches high values, intensive heating of lubricants occurs, which leads to a decrease in their viscosity and subsequent spontaneous leakage from the friction zone. This negatively affects the processes of lubrication and cooling of working surfaces, since the ability of the lubricant to form a stable protective film decreases, and the risk of overheating of the contacting elements increases. As a result, wear processes are intensified and the durability of hinged units is reduced. To prevent such undesirable phenomena, it is proposed to use special sealing devices that are capable of simultaneously performing several important functions: to protect the hinged joints of manipulators from the penetration of dust, moisture and aggressive impurities that can cause corrosive destruction of surfaces; to keep the lubricant in the friction zone, preventing its premature leakage. The conducted studies have confirmed the feasibility of integrating contact and labyrinth sealing elements into the design of hinges, which, due to their design specifics, provide more reliable protection of units from the negative effects of the operating environment and increase the service life of manipulators.

In the article [12], the method of synthesis of the trajectory of the manipulation robot movement taking into account its degrees of mobility is considered. It is shown that the bending of the rod causes support reactions in the contact zone, similar to the beam on two supports. Based on the determined contact pressure, it is possible to estimate the potential wear processes of the surfaces of the hydraulic cylinder, rod and stuffing box. It is established that even in the conditions of complete absence of danger of loss of strength by the rod during bending, contact stresses, which reach approximately one third of the material strength limit, can significantly accelerate the wear of friction surfaces. This approach allows to more accurately explain the reasons for the formation of characteristic wear patterns and to determine the features of their identification.

In the paper [13], it is shown that when developing new promising designs of articulated joints, it is necessary to apply a comprehensive approach to the selection of scientific and technical solutions, since their performance is influenced by a significant number of parameters. Such an approach creates opportunities for the formation of new design solutions that can provide increased reliability and durability of articulated joints of manipulators of logging machines. The implementation of such solutions allows to significantly improve both mechanical and tribotechnical characteristics of joints, as well as to optimize the thermal mode of their operation, which is an important factor in increasing the efficiency of equipment operation.

The article [14] presents the results of the analysis of the design features of the grippers of the body garbage trucks and the study of their reliability. Based on the conducted research, a calculation model of the garbage truck was developed, which was considered as an oscillatory system. During the analysis process, the features of the oscillations of the garbage truck frame during operation were established, and the dependencies of the formation of forces in the interaction of the elements of the "grip – tank – gripper" system were also revealed. The research showed that the greatest loads correspond to the traction and rod of the hydraulic cylinder, and these loads increase with increasing of container mass. At the same time, a change in the mass of the garbage truck itself does not affect the magnitude or amplitude of the loads, but leads to a change in their frequency characteristic. During operational observations, it was established that the main cause of garbage truck failures is wear and corrosion of the working surfaces of the equipment parts. In particular, 32% of all failures in the hydraulic drive system occur on hydraulic cylinders. Failures of these units are associated with wear of the working surfaces of the mating parts, deformations of the rod and cylinder under the action of operational loads arising from uneven loading of the body, as well as abrasive wear in difficult operating conditions. The main factor in failures of the hydraulic drive is intensive wear of the working surfaces of key parts of its design, in particular spools and housings of hydraulic distributors, as well as hydraulic cylinder rods. An additional factor of degradation is hydroabrasive damage, which occurs as a result of untimely replacement of the working hydraulic fluid and the use of poor-quality or worn sealing elements, such as hydraulic cylinder seals. This leads to the penetration of dust particles and wear products into the sliding zone, which significantly accelerates the destruction of the working surfaces. To increase the service life and restore the performance of parts, it is recommended to use the technology of chrome plating in cold self-regulating electrolyte, which ensures the production of chrome coatings with high quality of the deposit, increased wear resistance and sufficient productivity, which makes it one of the most promising methods of restoring worn elements of hydraulic drives.

Analytical study of the mathematical model of grinding of polymer waste in the grinding chamber of a rotary crusher with continuous classification of the finished product, carried out in [15], made it possible to determine with high accuracy the particle sizes of the final product, the crusher productivity, and energy consumption at different values of the rotor angular rotation speed, the initial waste sizes, the crusher design parameters, and the chamber loading modes.

In the article [16], an algorithm for numerical-analytical study of the dynamics of a planar six-bar linkage mechanism of a sewing machine thread take-up mechanism is proposed, which is based on the numerical solution of the differential equation of motion of the mechanism, and computer modeling of this mechanism is also carried out in Mathcad software.

In the paper [17], on the basis of analytical study of a mathematical model, the dependencies of the functioning of vibration and vibro-impact machines based on a hydropulse drive with a single-stage pulsator valve were determined.

In the materials of the article [18], an improved nonlinear mathematical model of the hydraulic drive operation of the mechanism for loading solid waste into a garbage truck during container overturning is proposed, which takes into account the wear of friction pairs and allowed to numerically study the dynamics of this drive during start-up and determine that taking into account the wear of friction pairs significantly affects the main parameters of the hydraulic drive for container overturning during MSW loading into a garbage truck. It was established that the duration of container overturning during MSW loading increases in a power-law dependence with increasing wear of the hydraulic cylinder.

The work [19] is dedicated to the determination of analytical dependencies that describe the quality indicators of transient processes of the hydraulic drive, which ensures the overturning of the container during the MSW loading into the garbage truck. In the article [20], a linearized mathematical model of the hydraulic drive for overturning the container during the technological operation of loading MSW into the garbage truck is proposed and analytically studied. However, this mathematical model does not take into account the wear of friction pairs.

However, in the process of analyzing known publications, authors did not find a linearized improved mathematical model of the hydraulic drive of the mechanism for overturning a MSW container into a garbage truck, taking into account the wear of friction pairs and the results of its analytical study.

Aims of the article

Analytical study of a linearized improved mathematical model of a hydraulic drive for a solid waste container overturning mechanism in garbage truck, taking into account the wear of friction pairs to obtain analytical dependencies of the main power and kinematic characteristics of this hydraulic drive in a steady-state mode of operation, which can be used when conducting design calculations of new garbage truck designs, taking into account the wear of the working bodies.

Methods

During the analytical study of the linearized improved mathematical model of the hydraulic drive of the mechanism for overturning a container with MSW in garbage truck, taking into account the wear of friction pairs, the following methods were used: operator calculus using the Laplace transformation to solve a system of ordinary linear differential equations, linearization of nonlinear dependencies, decomposition of complex expressions into simpler fractions, as well as computer modeling methods. To plot the graphs, the computer program "MatModel" described in [19] was used, which implements the 4th-order Runge-Kutta-Felberg numerical method with a variable integration step.

Results

Fig. 1 shows a calculation diagram of a linearized improved mathematical model of a hydraulic drive for overturning a container during the technological operation of MSW loading into a garbage truck, taking into account the wear of friction pairs when using the rear scheme of MSW loading. The diagram shows the following structural elements: C – container, G – gripper, L – lever, HC – hydraulic cylinder, HD – hydraulic distributor, P – hydraulic pump, SV – safety valve, F – filter, T – tank with working fluid, as well as the main geometric, kinematic and power parameters: p_1 , p_2 , p_3 , p_4 – pressures at the pump outlet, at the hydraulic cylinder inlet, at the hydraulic cylinder outlet and at the filter inlet, respectively; W_1 , W_2 , W_3 , W_4 – volumes of pipelines between the pump and the hydraulic distributor, the hydraulic distributor and the hydraulic cylinder inlet, the hydraulic cylinder outlet and the hydraulic distributor, the hydraulic distributor and the filter; Q_P –actual pump flow rate; S_P – cross-sectional area of the distributor opening; S_f – surface area of the filter element; D , d – piston and rod diameters; J – moment of inertia of moving elements; G_C –container weight; R – radius of rotation of moving elements; l_P – distance between the centers of rotation of the gripper and the rod; h – the height of the container; α – angle between the axes of the lever and the cylinder arm, γ – angle that takes into account the deviation of the center of mass position; δ – angle between the gripping arm and the horizontal; λ – angle of inclination of the container wall; ψ – the angle between the axis of the cylinder arm and the axis passing between the centers of rotation of the gripper and the hydraulic cylinder; φ – grip rotation angle.

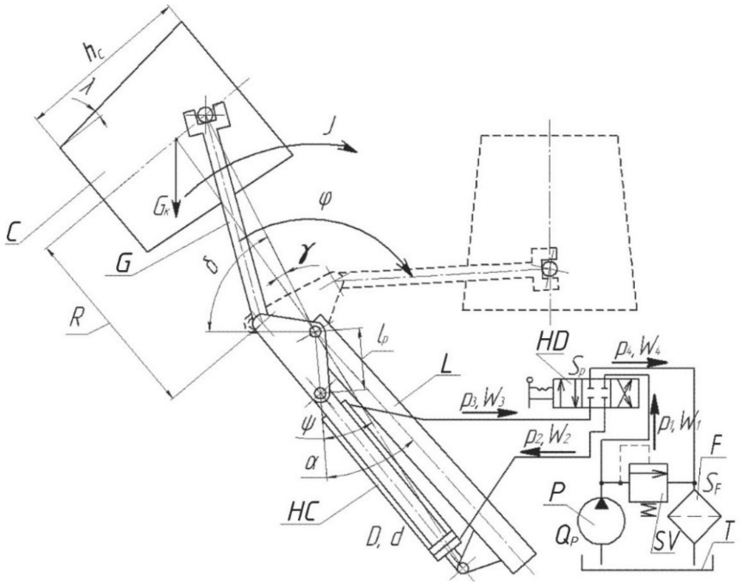


Fig. 1. Calculation scheme of the linearized improved mathematical model of the hydraulic drive for overturning the container during the technological operation of MSW loading, taking into account the wear of friction pairs

Analysis of the conducted studies of the nonlinear mathematical model [18] showed that $p_1 \approx p_2 \approx p_{12}$, and the influence of pressure in the drain lines and viscous friction forces on the operation of the hydraulic drive is insignificant. The process of overturning the container during the technological operation of MSW loading into a garbage truck can be divided into two phases:

1. The phase of overturning the container to the equilibrium position ($\delta+\varphi-\lambda \le \pi/2$).
2. Phase of pouring MSW from the container into the body of the garbage truck ($\delta+\varphi-\lambda > \pi/2$).

Thus, the system of differential equations of the nonlinear mathematical model is transformed into such systems of differential equations that correspond to the phases adopted above. The phase of rotation of the container to the equilibrium position can be described by a system of differential equations:

$$\begin{cases} Q_H = 2\dfrac{d\varphi}{dt} S_{Ц1} l_P \sin(\varphi+\psi) + \sigma_0 p_{12} + \alpha_\sigma\left(1-e^{-\beta_\sigma t}\right) + KW_{12}\dfrac{dp_{12}}{dt}; & (1) \\ p_{12} S_{Ц1} l_P \sin(\varphi+\alpha) = J\dfrac{d^2\varphi}{dt^2} + GR\cos(\varphi+\delta-\gamma), & (2) \end{cases}$$

where $W_{12} = W_1 + W_2$; σ_0 – coefficient of working fluid losses due to flow from the high-pressure region to the low-pressure region, $m^5/(N\cdot sec)$; α_σ, β_σ – coefficients of approximation of the dependence of working fluid losses on the duration of the friction pair wear process ($\alpha_\sigma = 4.054\cdot 10^{-4}$; $\beta_\sigma = 128.1$).

In order to linearize the trigonometric functions of the system of differential equations, let's make the following substitutions:

$$\cos(\varphi+\delta-\gamma) \approx \cos(\omega_0 t+\delta-\gamma); \quad (3)$$

$$\sin(\varphi+\alpha) \approx \sin(\overline{\varphi}_1+\alpha); \quad (4)$$

$$\sin(\varphi+\psi) \approx \sin(\overline{\varphi}_1+\psi), \quad (5)$$

$$\omega_0 \approx \frac{Q_P}{2S_{C1}R} = const, \quad (6)$$

where ω_0 – the averaged value of the angular velocity of the container in the 1st approximation, rad/sec; $S_{C1} = \pi\left(D^2-d^2\right)/4$ – area of the rod cavity of the hydraulic cylinder, m^2; $\overline{\varphi}_1 = (\pi/2+\lambda-\delta)/2$ – averaged in the first approximation value of the container overturning angle for the first phase, rad.

Therefore, the linearized mathematical model of the hydraulic drive of the first phase of container overturning for the operation of MSW loading into a garbage truck has the following form:

$$\begin{cases} Q_H = 2\omega S_{C1} l_p \sin(\overline{\varphi}_1 + \psi) + \sigma_0 p_{12} + \alpha_\sigma \left(1 - e^{-\beta_\sigma t}\right) + KW_{12} \dfrac{dp_{12}}{dt}; & (7) \\ p_{12} S_{C1} l_p \sin(\overline{\varphi}_1 + \alpha) = J \dfrac{d\omega}{dt} + GR\cos(\omega_0 t)\cos(\delta - \gamma) - GR\sin(\omega_0 t)\sin(\delta - \gamma), & (8) \end{cases}$$

where $\omega = \dfrac{d\varphi}{dt} \neq const$ – instantaneous value of the angular velocity of container overturning, rad/sec.

For further study of the linearized mathematical model, we will use the Laplace transformation, according to which we obtain the following:

$$\begin{cases} \dfrac{Q_P}{s} = \Omega(s) 2 S_{C1} l_p \sin(\overline{\varphi}_1 + \psi) + P(s)\sigma_0 + \dfrac{\alpha_\sigma}{s} - \dfrac{\alpha_\sigma}{s + \beta_\sigma} + P(s) s KW_{12}; & (9) \\ P(s) S_{C1} l_p \sin(\overline{\varphi}_1 + \alpha) = \Omega(s) s J + \dfrac{sGR\cos(\delta - \gamma)}{s^2 + \omega_0^2} - \dfrac{\omega_0 GR\sin(\delta - \gamma)}{s^2 + \omega_0^2}. & (10) \end{cases}$$

Substituting equation (10) into equation (9), we obtain:

$$\Omega_1(s) = \frac{-b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s(s + \beta_\sigma)(s^2 + \omega_0^2)(a_2 s^2 + a_1 s + a_0)}, \tag{11}$$

where $a_2 = KW_{12} J$; $a_1 = \sigma_0 J$; $a_0 = 2S_{C1}^2 l_p^2 \sin(\overline{\varphi}_1 + \alpha)\sin(\overline{\varphi}_1 + \psi)$; $b_4 = KW_{12} GR\cos(\delta - \gamma)$;
$b_3 = Q_P S_{C1} l_p \sin(\overline{\varphi}_1 + \alpha) - (KW_{12}\beta_\sigma + \sigma_0) GR\cos(\delta - \gamma) + KW_{12}\omega_0 GR\sin(\delta - \gamma)$;
$b_2 = (Q_P - \alpha_\sigma)\beta_\sigma S_{C1} l_p \sin(\overline{\varphi}_1 + \alpha) - \beta_\sigma \sigma_0 \omega_0 GR\sin(\delta - \gamma) + (KW_{12}\beta_\sigma + \sigma_0)\omega_0 GR\sin(\delta - \gamma)$;
$b_1 = Q_P \omega_0^2 S_{C1} l_p \sin(\overline{\varphi}_1 + \alpha) + \beta_\sigma \sigma_0 \omega_0 GR\sin(\delta - \gamma)$. $b_0 = (Q_P - \alpha_\sigma)\beta_\sigma \omega_0^2 S_{C1} l_p \sin(\overline{\varphi}_1 + \alpha)$ (12)

By the method of decomposing expression (11) into simpler fractions after reduction to the canonical form, we obtain

$$\begin{aligned} \Omega_1(s) = {} & A_1 \frac{1}{s} + F_1 \frac{1}{s + \beta_\sigma} + B_1 \frac{s}{s^2 + \omega_0^2} + \frac{D_1}{a_2} \frac{s + a_1/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0 a_2 - a_1^2)/(4a_2^2)} + \\ & + \frac{C_1}{\omega_0} \frac{\omega_0}{s^2 + \omega_0^2} + \frac{4E_1 - D_1 a_1}{2\sqrt{4a_0 a_2 - a_1^2}} \frac{\sqrt{4a_0 a_2 - a_1^2}/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0 a_2 - a_1^2)/(4a_2^2)}, \end{aligned} \tag{13}$$

where $A_1 = \dfrac{b_0}{\beta_\sigma a_0 \omega_0^2}$; $E_1 = -b_4 - (A_1 + B_1)a_1 + F_1(\beta_\sigma a_2 - a_1) - C_1 a_2$; $D_1 = -a_2(A_1 + F_1 + B_1)$;

$$C_1 = \frac{J_1 - A_1 a_0 \omega_0^2 (a_0 + \beta_\sigma a_1 - a_2\omega_0^2)}{G_1 \beta_\sigma} - F_1 \frac{\omega_0^2 (\beta_\sigma^2 a_1 a_2 - \beta_\sigma a_1^2 + a_0 a_1 + \beta_\sigma a_0 - \beta_\sigma a_2 \omega_0^2)}{G_1};$$

$$B_1 = -\frac{b_1}{\beta_\sigma a_1 \omega_0^2} + A_1 \frac{a_0}{\beta_\sigma a_1} + C_1 \frac{a_0 - \omega_0^2 a_2}{\omega_0^2 a_1} + F_1 \frac{\beta_\sigma a_2}{a_1} - \frac{b_4}{a_1}; \; I_1 = \beta_\sigma a_0^2 - \omega_0^2(\beta_\sigma a_0^2 + a_1^2 - 2\beta_\sigma a_0 a_2);$$

$$G_1 = a_1^2 \omega_0^2 + (a_0 - a_2\omega_0^2); \; H_1 = \beta_\sigma a_0 - \omega_0^2(\beta_\sigma a_2 + a_1); J_1 = \beta_\sigma a_1 \omega_0^2 (b_3 + \beta_\sigma b_4) + (a_0 - a_2\omega_0^2)(b_1 + \beta_\sigma \omega_0^2 b_4)$$

$$\begin{aligned} F_1 = {} & \frac{I_1 J_1 - A_1 a_0 \omega_0^2 \{I_1(a_0 + \beta_\sigma a_1 - a_2\omega_0^2) + G_1[\beta_\sigma(a_0 + \beta_\sigma a_1) - \omega_0^2(a_1 + \beta_\sigma a_2)]\} -}{\beta_\sigma \omega_0^2 \{H_1 G_1 \beta_\sigma a_2 - I_1[\beta_\sigma(\beta_\sigma a_1 a_2 - a_1^2 + a_0 - a_2\omega_0^2) + a_0 a_1]\}} \cdots \to \\ & \cdots \to \frac{- G_1 \{\beta_\sigma \omega_0^2 [a_1 b_2 - \beta_\sigma b_4 (a_0 - a_2\omega_0^2)] + H_1 b_1\}}{}. \end{aligned} \tag{14}$$

Then we can find the original image of (13):

$$\omega_1(t) = A_1 + F_1 e^{-\beta_\sigma t} + B_1 \cos(\omega_0 t) + \frac{C_1}{\omega_0}\sin(\omega_0 t) + \frac{D_1}{a_2} e^{-\frac{a_1}{2a_2}t} \cos\left(\frac{\sqrt{4a_0 a_2 - a_1^2}}{2a_2} t\right) + \\ + \frac{4E_1 - D_1 a_1}{2\sqrt{4a_0 a_2 - a_1^2}} e^{-\frac{a_1}{2a_2}t} \sin\left(\frac{\sqrt{4a_0 a_2 - a_1^2}}{2a_2} t\right). \tag{15}$$

Excluding insignificant coefficients in expression (15), which have a higher order of smallness, and taking into account the accepted notations according to (6), (12), (14), the angular velocity of container overturning, taking into account the wear of friction pairs during the first phase, is described by the following equation:

$$\omega_1(t) \approx \frac{Q_и - \alpha_\sigma}{2S_{C1}^2 l_p^2 \sin(\overline{\varphi}_1 + \psi)}\left[1 + \frac{2S_{C1}^2 l_p^2 \sin(\overline{\varphi}_1 + \alpha)\sin(\overline{\varphi}_1 + \psi)}{\beta_\sigma \sigma_0 J}\right] \times \\ \times \left\{1 - e^{-\frac{\sigma_0}{2KW_{12}}t} \cos\left[S_{C1} l_p \sqrt{\frac{2\sin(\overline{\varphi}_1 + \alpha)\sin(\overline{\varphi}_1 + \psi)}{KW_{12} J}t}\right]\right\}. \tag{16}$$

After substituting the substitution $\sin(\overline{\varphi} + \alpha)\sin(\overline{\varphi} + \psi) \approx \sin^2[\overline{\varphi} + (\alpha + \psi)/2]$ into equation (16):

$$\omega_1(t) \approx \frac{Q_и - \alpha_\sigma}{2S_{C1}^2 l_p^2 \sin(\overline{\varphi}_1 + \psi)}\left[1 + \frac{2S_{C1}^2 l_p^2 \sin^2\left(\overline{\varphi}_1 + \frac{\alpha + \psi}{2}\right)}{\beta_\sigma \sigma_0 J}\right]\left\{1 - e^{-\frac{\sigma_0}{2KW_{12}}t} \cos\left[S_{C1} l_p \sin\left(\overline{\varphi}_1 + \frac{\alpha + \psi}{2}\right)\sqrt{\frac{2}{KW_{12} J}t}\right]\right\}. \tag{17}$$

To determine the container overturning angle, taking into account the wear of friction pairs during the first phase, we integrate equation (17) and, taking into account the initial conditions $\varphi(0) = 0$, we obtain the following:

$$\varphi_1(t) = \frac{Q_и - \alpha_\sigma}{2S_{C1}^2 l_p^2 \sin(\overline{\varphi}_1 + \psi)}\left[1 + \frac{2S_{C1}^2 l_p^2 \sin^2\left(\overline{\varphi}_1 + \frac{\alpha + \psi}{2}\right)}{\beta_\sigma \sigma_0 J}\right]\left\langle\left\{\frac{e^{-\frac{\sigma_0}{2KW_{12}}t}}{8KW_{12} S_{C1}^2 l_p^2 \sin^2[\overline{\varphi}_1 + (\alpha + \psi)/2] + J\sigma_0^2} \times \right.\right. \\ \times \left\{2KW_{12} J\sigma_0 \cos\left[S_{Ц1} l_p \sin\left(\overline{\varphi}_1 + \frac{\alpha + \psi}{2}\right)\sqrt{\frac{2}{KW_{12} J}t}\right] - 4K^{1,5} W_{12}^{1,5} S_{C1} l_p \sqrt{2J} \times \right. \\ \left.\left. \times \sin\left(\overline{\varphi}_1 + \frac{\alpha + \psi}{2}\right) \sin\left[S_{C1} l_p \sin\left(\overline{\varphi}_1 + \frac{\alpha + \psi}{2}\right)\sqrt{\frac{2}{KW_{12} J}t}\right]\right\} + t\right\rangle. \tag{18}$$

Excluding insignificant coefficients in expression (18), which have a higher order of smallness, we obtain a simplified equation for the change in the container overturning angle taking into account the wear of friction pairs during the first phase:

$$\varphi_1(t) \approx \frac{Q_и - \alpha_\sigma}{2S_{C1}^2 l_p^2 \sin(\overline{\varphi}_1 + \psi)}\left\{1 + \frac{2S_{C1}^2 l_p^2 \sin^2[\overline{\varphi}_1 + (\alpha + \psi)/2]}{\beta_\sigma \sigma_0 J}\right\}t. \tag{19}$$

From equation (19), we determine the duration of container overturning, taking into account the wear of friction pairs during the first phase:

$$t_1 \approx \frac{2S_{C1}^2 l_p^2 \beta_\sigma \sigma_0 J \sin(\overline{\varphi}_1 + \psi)}{(Q_P - \alpha_\sigma)\{\beta_\sigma \sigma_0 J + 2S_{C1}^2 l_p^2 \sin^2[\overline{\varphi}_1 + (\alpha + \psi)/2]\}}\varphi_1. \tag{20}$$

Solving the system of equations (9, 10) with respect to $P(s)$ after reduction to the canonical form, we obtain:

$$P_1(s)=A_{1p}\frac{1}{s}+I_{1p}\frac{1}{s+\beta_\sigma}+\frac{B_{1p}+J_{1p}-C_{1p}-F_{1p}}{KW_{12}}\frac{1}{s^2+\sigma_0/(KW_{12})}-D_{1p}\frac{s}{s^2+\omega_0^2}-\frac{E_{1p}}{\omega_0}\frac{\omega_0}{s^2+\omega_0^2}-$$
$$-\frac{G_{1p}}{a_2}\frac{s+a_1/(2a_2)}{[s+a_1/(2a_2)]^2+(4a_0a_2-a_1^2)/(4a_2^2)}-\frac{4H_{1p}-G_{1p}a_1}{2\sqrt{4a_0a_2-a_1^2}}\frac{\sqrt{4a_0a_2-a_1^2}/(2a_2)}{[s+a_1/(2a_2)]^2+(4a_0a_2-a_1^2)/(4a_2^2)}, \tag{21}$$

where $A_{1p}=\left[Q_P-\alpha_\sigma-2A_1S_{C1}l_p\sin(\overline{\varphi}_1+\psi)\right]/\sigma_0$; $B_{1p}=-KW_{12}\left[Q_P-\alpha_\sigma-2A_1S_{C1}l_p\sin(\overline{\varphi}_1+\psi)\right]/\sigma_0$;

$$C_{1p}=-\frac{2S_{C1}l_pKW_{12}(B_1\sigma_0-C_1KW_{12})\sin(\overline{\varphi}_1+\psi)}{\sigma_0^2+K^2W_{12}^2\omega_0^2};\ D_{1p}=\frac{2S_{C1}l_p(B_1\sigma_0-C_1KW_{12})\sin(\overline{\varphi}_1+\psi)}{\sigma_0^2+K^2W_{12}^2\omega_0^2};$$
$$E_{1p}=\frac{2S_{C1}l_p(B_1KW_{12}\omega_0^2+C_1\sigma_0)\sin(\overline{\varphi}_1+\psi)}{\sigma_0^2+K^2W_{12}^2\omega_0^2};\ H_{1p}=\frac{2S_{C1}l_p(E_1(a_1KW_{12}-a_2)-D_1a_0KW_{12})\sin(\overline{\varphi}_1+\psi)}{a_1KW_{12}\sigma_0-a_2\sigma_0^2-a_0K^2W_{12}^2};$$
$$F_{1p}=\frac{2S_{C1}l_pKW_{12}(D_1\sigma_0-E_1KW_{12})\sin(\overline{\varphi}_1+\psi)}{a_1KW_{12}\sigma_0-a_2\sigma_0^2-a_0K^2W_{12}^2};\ G_{1p}=-\frac{2S_{Ц1}l_pa_2(D_1\sigma_0-E_1KW_{12})\sin(\overline{\varphi}_1+\psi)}{a_1KW_{12}\sigma_0-a_2\sigma_0^2-a_0K^2W_{12}^2};$$
$$I_{1p}=\frac{\alpha_\sigma-2F_1S_{C1}l_p\sin(\overline{\varphi}_1+\psi)}{\sigma_0-KW_{12}\beta_\sigma}.\ J_{1p}=\frac{KW_{12}\left[\alpha_\sigma-2F_1S_{C1}l_p\sin(\overline{\varphi}_1+\psi)\right]}{KW_{12}\beta_\sigma-\sigma_0} \tag{22}$$

Next, we can find the original image of (21)

$$p_1(t)=A_{1p}+I_{1p}e^{-\beta_\sigma t}+\frac{B_{1p}+J_{1p}-C_{1p}-F_{1p}}{KW_{12}}e^{-\frac{\sigma_0}{KW_{12}}t}-D_{1p}\cos(\omega_0t)-\frac{E_{1p}}{\omega_0}\sin(\omega_0t)-$$
$$-\frac{G_{1p}}{a_2}e^{-\frac{a_1}{2a_2}t}\cos\left(\frac{\sqrt{4a_0a_2-a_1^2}}{2a_2}t\right)-\frac{4H_{1p}-G_{1p}a_1}{2\sqrt{4a_0a_2-a_1^2}}e^{-\frac{a_1}{2a_2}t}\sin\left(\frac{\sqrt{4a_0a_2-a_1^2}}{2a_2}t\right). \tag{23}$$

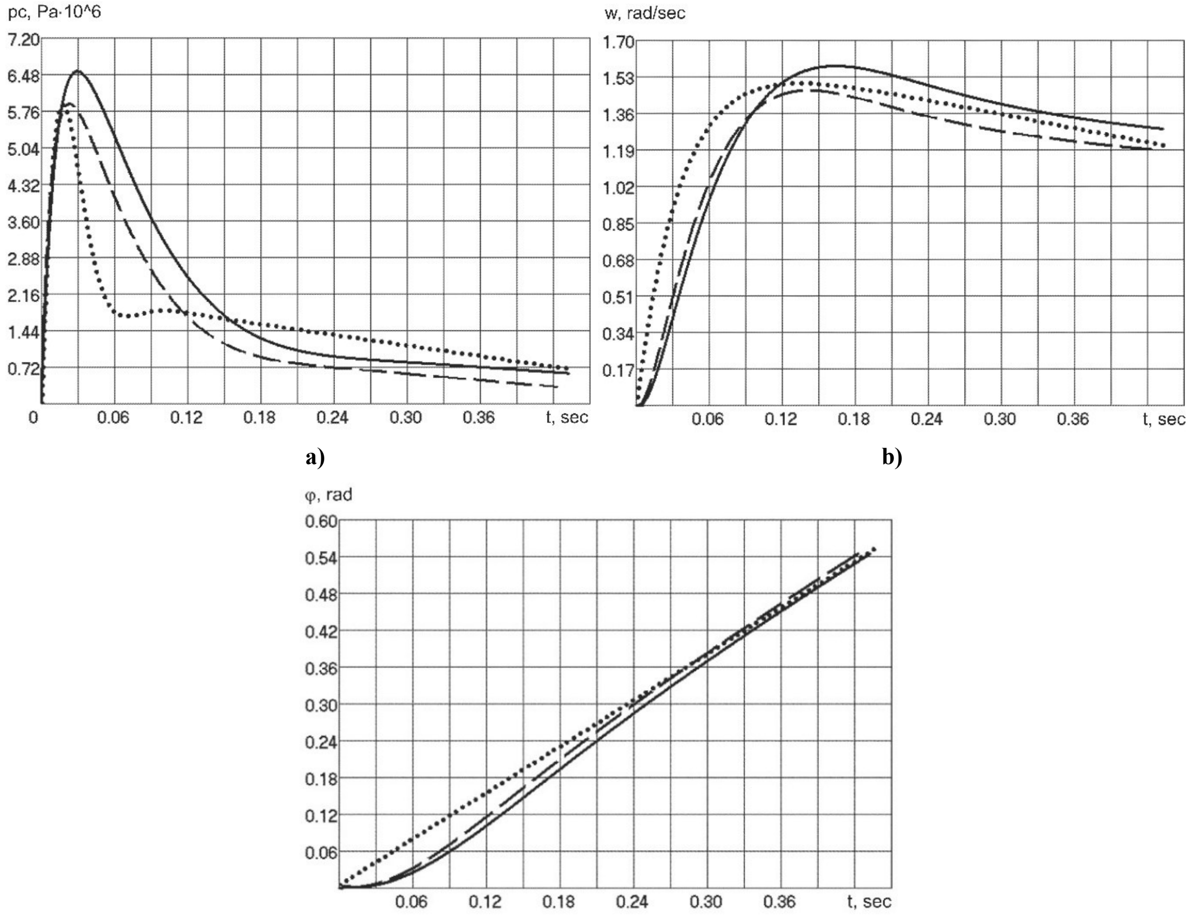
Neglecting the insignificant coefficients of equation (23), which have a higher order of smallness, and taking into account the notations according to (6), (12), (14), (22) and the initial conditions $p(0) = 0$, the pressure in the hydraulic cylinder pressure line, taking into account the wear of the friction pairs during the first phase, can be described by the following equation:

$$p_1(t)\approx\frac{\alpha_\sigma}{\sigma_0-KW_{12}\beta_\sigma}e^{-\beta_\sigma t}+\frac{GR\cos(\delta-\gamma)}{S_{C1}l_p\sin(\overline{\varphi}_1+\alpha)}+\frac{Q_P}{S_{C1}l_p\sin\left(\overline{\varphi}_1+\frac{\alpha+\psi}{2}\right)}\sqrt{\frac{J}{2KW_{12}}}e^{-\frac{\sigma_0}{2KW_{12}}t}\sin\left\langle\sqrt{\frac{2}{KW_{12}J}}S_{C1}\times\right.$$
$$\left.\times l_p\sin\left(\overline{\varphi}_1+\frac{\alpha+\psi}{2}\right)t-\arcsin\left\{\left[\frac{\alpha_\sigma}{\sigma_0-KW_{12}\beta_\sigma}+\frac{GR\cos(\delta-\gamma)}{S_{C1}l_p\sin(\overline{\varphi}_1+\alpha)}\right]\frac{S_{C1}l_p\sin[\overline{\varphi}_1+(\alpha+\psi)/2]}{Q_P}\sqrt{\frac{2KW_{12}}{J}}\right\}\right\rangle. \tag{24}$$

Comparison of the results obtained using nonlinear and linearized mathematical models of the hydraulic drive for overturning the container during the technological operation of loading MSW into a garbage truck, taking into account the wear in friction pairs, as well as using equations obtained as a result of the analytical solution of the linearized model, is shown in Fig. 2.

When comparing the characteristics of container overturning obtained using a nonlinear mathematical model and equations (16), (19), (24), as a result of analytical solution of the linearized mathematical model of the hydraulic drive for container overturning in the technological operation of MSW loading into a garbage truck, taking into account the wear of friction pairs, the error is about 10% at the beginning of the movement and decreases to 2-5% at the end of the movement compared to the nonlinear mathematical model, which is acceptable for performing preliminary design calculations. If necessary, the values of the main parameters can be specified at the final stage of design using a nonlinear mathematical model.

The obtained regression equation (20) allows to approximately determine the duration of the 1st phase – the rotation of the container to the equilibrium position during the container overturning during the technological operation of MSW loading into a garbage truck, taking into account the wear of friction pairs, which can be used when conducting design calculations of new garbage truck designs, taking into account the wear of working bodies without the need to study the nonlinear mathematical model of the drive, as well as when optimizing the main parameters of the hydraulic drive.



c)

Fig. 2. Comparison of results obtained using nonlinear (——) and linearized (– – –) mathematical models of the hydraulic drive for overturning the container during the technological operation of MSW loading into a garbage truck, taking into account the wear of friction pairs, as well as using the equations obtained as a result of its analytical solution (···): a) –change in pressure in the hydraulic cylinder; b) –change in angular velocity; c) –angle of rotation

Analytical study of the linearized improved mathematical model of the hydraulic drive of the mechanism for MSW loading into a garbage truck, taking into account the wear of friction pairs during the 2nd phase – pouring solid waste from the container into the body of the garbage truck, requires further research.

Conclusions

To carry out design calculations of new garbage truck designs, approximate analytical dependences of the pressure in the hydraulic cylinder pressure line, the angular velocity and the angle of container overturning on time were obtained. It was based on the proposed linearized mathematical model of the hydraulic drive of the container overturning mechanism in the technological operation of loading municipal solid waste into the garbage truck during the 1st phase: overturning the container to the equilibrium position, taking into account the wear of friction pairs. A regression equation was obtained, which allows to approximately determine the duration of the 1st phase – the rotation of the container to the equilibrium position during its overturning during the technological operation of loading of municipal solid waste (MSW) into a garbage truck, taking into account the wear of friction pairs, which can be used during design calculations of new garbage truck designs taking into account the wear of working bodies without the need to study the nonlinear mathematical model of the drive, as well as during optimization of the main parameters of the hydraulic drive. It was established that the analytical study of the linearized improved mathematical model of the hydraulic drive of the mechanism for MSW loading into a garbage truck, taking into account the wear of friction pairs during the 2nd phase – pouring waste from the container into the body of the garbage truck, requires further research.

References

1. Holenko K., Dykha O., Koda E., Kernytskyy I., Horbay O., Royko Y., Fornalchyk Y., Berezovetska O., Rys V., Humenuyk R., Berezovetskyi S., Żółtowski M., Baryłka A., Markiewicz A., Wierzbicki T., Bayat H. (2024) Structure and strength optimization of the Bogdan ERCV27 electric garbage truck spatial frame under static loading. Applied Sciences, 14(23), 11012, https://doi.org/10.3390/app142311012

2. Kindrachuk M.V., Kharchenko V.V., Marchuk V.Y., Humeniuk I.A., Leusenko D.V. (2024) Methodology for Selecting Compatible Metal Materials for Friction Pairs During Fretting-Corrosion Wear. Metallophysics & Advanced Technologies, 46(7), 637-648, https://doi.org/10.15407/mfint.46.07.0637

3. Dykha A., Sorokatyi R., Pasichnyk O., Yaroshenko P., Skrypnyk T. (2020, December) Machine wear calculation module in computer-aided design systems. In IOP Conference Series: Materials Science and Engineering, 1001(1), 012040, https://doi.org/10.1088/1757-899X/1001/1/012040

4. Petrov O., Kozlov L., Lozinskiy D., Piontkevych O. (2019) Improvement of the hydraulic units design based on CFD modeling. Lecture Notes in Mechanical Engineering, 653-660, https://doi.org/10.1007/978-3-030-22365-6_65

5. The Cabinet of Ministers of Ukraine (2004) Resolution No. 265 "Pro zatverdzhennia Prohramy povodzhennia z tverdymy pobutovymy vidkhodamy" ["On Approval of the Program for Solid Waste Management"]. URL: http://zakon1.rada.gov.ua/laws/show/265-2004-%D0%BF.

6. Lazea M., Constantin G.A., Stoica D., Voicu G. (2020) Analiza structurala cu elemente finite a pieselor de uzura de pe placa de contrapresiune a unei autogunoiere. Revista Romana de Materiale, 50(2), 283-293.

7. Qiu Z., Min R., Wang D., Fan S. (2022) Energy features fusion based hydraulic cylinder seal wear and internal leakage fault diagnosis method. Measurement, 195, 111042, https://doi.org/10.1016/j.measurement.2022.111042

8. Li H., Li Y. (2025) Finite element analysis and structural optimization design of multifunctional robotic arm for garbage truck. Frontiers in Mechanical Engineering, 11, 1543967, https://doi.org/10.3389/fmech.2025.1543967

9. Kot T., Bobovský Z., Vysocký A., Krys V., Šafařík J., Ružarovský R. (2021) Method for robot manipulator joint wear reduction by finding the optimal robot placement in a robotic cell. Applied Sciences, 11(12), 5398, https://doi.org/10.3390/app11125398

10. Chernik D.V., Chernik K.N. (2020) Mathematical model of a combined manipulator of a forest machine. In IOP Conference Series: Materials Science and Engineering, 5(919), 052037, https://doi.org/10.1088/1757-899X/919/5/052037

11. Serebryanskii A. (2014) Except the negative reverse effect and automatically compensate for wear in the hinge manipulators. DOAJ – Lund University: Concept: Scientific and Methodological e-magazine, 4 (Collected works, Best Article).

12. Beisembayev A., Yerbossynova A., Pavlenko P., Baibatshayev M. (2023) Planning trajectories of a manipulation robot with a spherical coordinate system for removing oxide film in the production of commercial lead, zinc. Eastern-European Journal of Enterprise Technologies, 124(2), 80, https://doi.org/10.15587/1729-4061.2023.286463

13. Zadorozhnaya E., Levanov I., Kandeva M. (2018, May) Tribological research of biodegradable lubricants for friction units of machines and mechanisms: Current state of research. In International Conference on Industrial Engineering, 939-947, https://doi.org/10.1007/978-3-319-95630-5_98

14. Kargin R.V., Yakovlev I.A., Shemshura E.A. (2017) Modeling of workflow in the grip-container-grip system of body garbage trucks. Procedia Engineering, 206, 1535-1539, https://doi.org/10.1016/j.proeng.2017.10.727

15. Skyba M.E., Misyats O.V., Polishchuk A.O., Misyats V.P., Rubanka M.M. (2021) Systema adaptyvnoho chastotnoho keruvannia shvydkistiu obertannia asynkhronnoho tryfaznoho elektrodvyhuna pryvodu rotornoi drobarky [Adaptive frequency control system for the rotation speed of an asynchronous three-phase electric motor of a rotary crusher drive]. Herald of Khmelnytskyi National University. Technical sciences, 2(295), 139-146, https://doi.org/10.31891/2307-5732-2021-295-2-139-146

16. Manoilenko O., Dvorzhak V., Horobets V., Panasiuk I., Bezuhlyi D. (2024) Assessing the impact of sewing machine thread take-up mechanism parameters on the magnitude and nature of thread take-up. Eastern European Journal of Enterprise Technologies, 2024, 6(1(132)), 64-75, https://doi.org/10.15587/1729-4061.2024.315129

17. Iskovich-Lototsky R., Kots I., Ivanchuk Y., Ivashko Y., Gromaszek K., Mussabekova A., Kalimoldayev M. (2019). Terms of the stability for the control valve of the hydraulic impulse drive of vibrating and vibro-impact machines. Przeglad Elektrotechniczny, 4(19), 19-23, https://doi.org/10.15199/48.2019.04.04

18. Berezyuk O.V., Yavorskyi V.Ye. (2025) Udoskonalena matematychna model roboty hidropryvodu mekhanizmu zavantazhennia tverdykh pobutovykh vidkhodiv u smittievoz iz urakhuvanniam znosu par tertia [Improved mathematical model of the hydraulic drive of the mechanism for loading solid household waste into a garbage truck taking into account the wear of friction pairs]. Scientific Works of Vinnytsia National Technical University, 1, 154-164, https://doi.org/10.31649/2307-5376-2025-1-154-164

19. Bereziuk O., Petrov O., Lemeshev M., Slabkyi A., Sukhorukov S. (2023) Transient Processes Quality Indicators of the Rotation Lever Hydraulic Drive for the Dust-Cart Manipulator. Lecture Notes in Mechanical Engineering, 2, 3-12, https://doi.org/10.1007/978-3-031-32774-2_1

20. Berezyuk O., Savulyak V. (2017) Approximated mathematical model of hydraulic drive of container upturning during loading of solid domestic wastes into a dustcart. Technical Sciences, 20(3), 259-273, https://doi.org/10.31648/ts.5426

Березюк О.В., Савуляк В.І., Харжевський В.О., Іванов С.Ц., Яворський В.Є. Аналітичне дослідження моделі гідроприводу механізму перевертання контейнера з побутовими відходами у сміттєвоз із урахуванням зносу пар тертя.

Стаття присвячена аналітичному дослідженню удосконаленої математичної моделі гідроприводу механізму перевертання контейнера з твердими побутовими відходами у сміттєвоз із урахуванням зносу пар тертя. У результаті аналізу виконаних числових досліджень нелінійної удосконаленої математичної моделі гідроприводу механізму перевертання контейнера з твердими побутовими відходами у сміттєвоз із урахуванням зносу пар тертя, розроблено лінеаризовану версію цієї моделі у формі системи звичайних лінійних диференціальних рівнянь другого порядку. Для проведення проектних розрахунків нових конструкцій сміттєвозів отримано наближені аналітичні залежності тиску в напірній магістралі гідроциліндра, кутової швидкості та кута перевертання контейнера від часу на основі запропонованої лінеаризованої математичної моделі гідроприводу механізму перевертання контейнера на технологічній операції завантаження твердих побутових відходів у сміттєвоз під час 1-ї фази – повороту контейнера до положення рівноваги із урахуванням зносу пар тертя. Одержане рівняння регресії, яке дозволяє наближено визначити тривалість 1-ї фази – повороту контейнера до положення рівноваги під час його перевертання на технологічній операції завантаження твердих побутових відходів у сміттєвоз із урахуванням зносу пар тертя, що може бути використано під час проведення проектних розрахунків нових конструкцій сміттєвозів із урахуванням зносу виконавчих органів без необхідності дослідження нелінійної математичної моделі приводу його робочих органів, а також під час оптимізації основних параметрів гідроприводу. Встановлено, що аналітичне дослідження лінеаризованої удосконаленої математичної моделі гідроприводу механізму завантаження твердих побутових відходів у сміттєвоз із урахуванням зносу пар тертя під час 2-ї фази – висипання відходів із контейнера в кузов сміттєвоза вимагає проведення подальших досліджень.

Ключові слова: лінеаризована математична модель, урахування зносу, перетворення за Лапласом, гідропривод, знос, вузли тертя, механізм перевертання контейнера, сміттєвоз, тверді побутові відходи