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Analytical study of an improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder

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Abstract

The article is dedicated to the analytical study of the improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, in which the influence of the wear of the hydraulic cylinder on its operation is additionally taken into account. On the basis of the numerical studies of the improved nonlinear mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, which takes into account the wear of the hydraulic cylinder, its linearized modification was created, presented in the form of a system of ordinary linear differential equations of the second order. To perform design calculations of new modifications of garbage trucks, approximate analytical expressions were obtained that describe the change in time of the pressure in the hydraulic cylinder's pressure line, as well as the speed and movement of the sealing plate based on the proposed linearized mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of the hydraulic cylinder. The obtained regression equation that provides the possibility of an approximate determination of the duration of the process of compacting municipal solid waste in a garbage truck, taking into account the wear of the hydraulic cylinder. This equation can be used during the design process of new garbage truck, allowing to take into account the change in the technical condition of the actuators without the need to analyze the full nonlinear mathematical model of the drive of the working bodies. In addition, it is suitable for the usage in the optimization procedures of the main parameters of the hydraulic drive. It was found that the creation of an improved methodology for design calculations of new garbage truck constructions, taking into account the wear of the actuators, requires additional scientific research and further development of the corresponding theoretical methodology.

Keywords: linearized mathematical model, consideration of wear, Laplace transformation, hydraulic drive, wear, hydraulic cylinder, sealing plate mechanism, garbage truck, municipal solid waste.

Introduction

One of the important directions of development of modern municipal engineering in Ukraine is to increase the efficiency and modernization of mobile equipment of garbage trucks, which determines the topicality of relevant scientific and engineering research [1]. In this area, it is so important to ensure high wear resistance, reliability and long service life of machine elements, because those parameters determine the efficiency of equipment operation. Increasing these characteristics helps to reduce maintenance and repair costs, reduce the number of equipment downtimes, and also extend its overall service life under conditions of significant and frequently repeated loads that are typical for intensive operation of municipal vehicles [2, 3]. In Ukraine, most of the work involved in collecting and transporting municipal solid waste (MSW) to landfills or processing plants is carried out using garbage trucks. An important element of such vehicles is the sealing plate mechanism, which is operated by a hydraulic drive [4, 5]. Today, Ukraine has about 3,700 garbage trucks in operation, which perform not only the function of transporting MSW, but also carry out their preliminary compaction. The use of such equipment makes it possible to significantly reduce the costs associated with MSW transportation, as well as reduce the required area of landfills for their disposal. This provides a significant economic effect and helps reduce the negative anthropogenic impact on the environment. During the compaction of MSW in a garbage truck, the highest loads are observed in the friction units, in particular in the hydraulic cylinder. Intensive wear of these elements is caused by several factors: significant weight of MSW in the body, the need for mechanisms to operate in reverse mode with repeated reciprocating movements, as well as high repetition of operating cycles during a single trip. An additional factor complicating operation is the influence of variable environmental conditions, in particular significant fluctuations in relative humidity and temperature, as well as a high level of their contamination. The combination of these factors causes accelerated wear of working surfaces, which negatively affects the reliability and durability of garbage trucks. Deterioration of the operational properties of the materials of parts or insufficient lubrication leads to an increase in friction forces in friction pairs. As a result, system vibrations increase, which negatively affects its dynamic stability and reduces the mechanism’s ability to withstand high loads during operation in reverse mode. Accelerated wear of friction units reduces the efficiency of the garbage truck and increases potential risks to the safety of its operation. This can cause emergency operating modes that threaten the health of operators and at the same time negatively affect the environment in the event of uncontrolled spillage or leakage of solid waste. According to the provisions of the Resolution of the Cabinet of Ministers of Ukraine No. 265 [6], one of the priority areas for the development of municipal services is the introduction of modern highly efficient garbage trucks. These machines play a key role in the system of technical means intended for collection, transportation and primary processing of solid waste. The introduction of modern models of garbage trucks allows not only to optimize logistics processes and reduce operating costs, but also contributes to a comprehensive solution to important environmental problems associated with waste management. In addition, the renewal of the fleet of equipment increases the overall reliability and efficiency of municipal services, which is of strategic importance for ensuring the sustainable development of settlements. Planning the renewal, maintenance and repair of garbage trucks is significantly facilitated by using approximate analytical dependencies of the main power and kinematic characteristics of the hydraulic drive of the garbage truck’s sealing plate mechanism, taking into account the wear of its hydraulic cylinder.

Analysis of recent research and publications

Vehicles that collect and transport solid waste are equipped with complex systems that perform the loading of waste from stationary collection containers, the reception and pre-compaction of the material, compaction in the body of the garbage truck and, finally, the disposal of the collected waste at a landfill. Of all these operations, the compaction process is the most mechanically demanding. In the article [7], a structural analysis of the compaction sealing plate of a garbage truck is presented. In the first stage, a parametric modeling of the assembly consisting of the sealing plate, the counter-pressure plate, the rear part of the garbage truck and the material to be compacted was carried out. Dynamic modeling of the MSW compaction process in a garbage truck was carried out, and the mechanical stresses for the compaction sealing plate were used into the modeling module of the SOLIDWORKS 2016 CAD/CAE software, with the help of which a finite element analysis was performed. As a result, the equivalent stresses distribution that were calculated using the von Mises criterion, displacements, and relative deformations of the sealing plate of the analyzed garbage truck were obtained.

In the scientific work [8], a detailed structural analysis of the main elements of a garbage truck that are subject to intensive wear during operation was carried out. The work considered the design features of these parts, the conditions of their operation and the factors that most significantly affect the speed and nature of the degradation of friction surfaces. Particular attention was paid to the definition of vulnerable zones that limit the durability of garbage truck mechanisms, as well as the systematization of typical manifestations of wear that occur under the influence of loads, operating modes and external operating conditions. In the study [9], the authors proposed an improved method for technical diagnostics of hydraulic cylinder malfunctions associated with wear of sealing elements and the appearance of internal leaks of the working fluid. The method is based on the analysis and integration of the energy characteristics of the hydraulic drive, which allows to increase the sensitivity of detecting defects at the early stages of their development. The usage of an approach based on the merging of energy parameters provides a more comprehensive assessment of the technical condition of the hydraulic cylinder, enables the detection of hidden signs of deviations in the system’s operation, and increases the accuracy of predicting its remaining service life.

The study [10] comprehensively analyzes the kinematics and dynamics of the scraping mechanism for compacting MSW in a garbage truck using numerical methods. To study the operation of the mechanism under actual working conditions during a full cycle lasting 18 seconds, a multi-body structure integrated with a hydraulic simulation model was created. The model was verified by calculation at stationary moments in time, which showed high consistency. The results show that the pressing mechanism operates in steady-state modes almost all the time, with cylinder speeds ranging from 0.08 to 0.15 m/sec. The speed and acceleration of the hydraulic cylinder fluctuate greatly when the mechanism accelerates or decelerates, but the effect of inertia is negligible. The forces applied to the joints are maximum at the end of the pressing process. It is notable that the force applied to the connection between the scraper plate and the sliding plate is the highest: three times higher than that applied to the connection between the sliding plate and the press cylinder, and one and a half times higher than that applied to the connection between the scraper plate and the scraper cylinder. The results of the study can be applied to the design process of garbage trucks in specialized vehicles, or used as a basis for improving productivity and optimizing the weight, strength, and materials of the mechanism.

The study [5] provides a detailed analysis of hydrodynamic processes occurring during the flow of working fluid through a water seal using computer modeling methods. By means of numerical experiments, the authors were able to establish the nature of hydraulic losses in local narrowings and channels of the water seal, as well as quantitatively estimate the level of pressure loss resulting from these processes. The results obtained made it possible to comprehensively characterize the interaction of fluid flows with the structural elements of the internal cavities of the water seal and to identify the most critical areas where an intense pressure drop occurs. Based on the analysis, a number of design improvements to the hydraulic lock were proposed, aimed to optimize the shape of its channels and improve fluid flow conditions. The proposed changes do not affect the functional purpose of this element, but ensure a more uniform distribution of velocities in the flow and a reduction in local turbulent zones. The implementation of such design solutions makes it possible to reduce pressure losses in the working part of the hydraulic distributor, which is an important factor in increasing the energy efficiency of the hydraulic drive. As a result, the overall energy losses in the system are reduced, which has a positive effect on its performance, stability, and durability.

Scientific publication [11] provides an in-depth study of the specifics of the wood chip pressing process in screw presses. The authors analyzed the sequence of physical and mechanical phenomena occurring in different functional areas of the screw during the movement and compaction of raw materials. Particular attention is paid to describing the behavior of wood chips under conditions of variable pressure and intense friction that arise in the areas of feeding, transportation, compaction, and briquette formation. The dependencies that were obtained in the study made it possible to establish methods for engineering calculations of the loads acting on the screw turns and to determine the dependencies that define the optimal design parameters of the screw press. In addition, based on theoretical analysis and experimental observations, the required drive power was determined to ensure a stable pressing process under various operating modes. The thermal processes accompanying the compaction of wood chips were studied separately. The dependencies of change in the temperature of raw materials along the length of the screw were established and the factors influencing the intensity of its heating were determined. The specific energy costs required for the formation of the pressed product were also calculated, which allows to increase the energy efficiency of the technological process and ensure a rational choice of operating modes for the pressing equipment.

The scientific article [12] establishes that the wear rate of the working hydraulic cylinder of the garbage truck's compactor sealing plate mechanism is exponentially dependent on the pressing force. Taking this dependency into account allows for the optimization of maintenance and repair planning, which, in turn, increases the overall efficiency of garbage truck operation. To illustrate this process, a graph was constructed showing the change in the wear rate of the hydraulic cylinder as a function of the pressing force, which confirmed the good correlation between the obtained model and the experimental data. In particular, for the serial Ukrainian garbage truck model KO-436, the wear rate of the working hydraulic cylinder according to this dependency is 0.257 µm/h. At the same time, increasing the pressing force from 30 MN to 150 MN reduces the wear rate of the hydraulic press cylinder by 3.6 times. This effect is explained by the specifics of contact processes between working surfaces and the peculiarities of the mechanism's operation at different load levels.

The analysis of the causes of typical technical failures of garbage truck units, presented in [13], shows that a significant percentage of malfunctions (about 45%) are happened due to hydraulic drive failures. The main factors contributing to these failures are manufacturing defects associated with the usage of low-quality components, as well as significant fluctuations in the loads on the working parts. A study of the causes of working body failures showed that the dominant factors are heat treatment defects and deviations from design parameters during mechanical processing (35%), mistakes in the assembly process, adjustment, and tightening of threaded connections (30%), and poor-quality welding (30%). It has been established that the majority of failures (80–90%) occur as a result of wear and corrosion of the working surfaces of parts, with failure occurring after a critical level of degradation has been reached, i.e., when the unit or machine reaches its limit technical condition. In particular, up to 28% of all failures of hydraulic drive elements occur in hydraulic cylinders, which is caused by wear of mating surfaces and deformation of the rod and cylinder during operation. Durability analysis shows that the average service life of hydraulic drive components, in particular hydraulic cylinders, is only about one-third of their maximum service life, i.e., the service life specified by the manufacturer is not achieved by 45-55%. The highest percentage of hydraulic cylinder failures in the early stages of operation or after repair occurs in the rods (31%) and sealing sleeves (42%). In addition, a common form of failure of hydraulic system components is the loss of external and internal tightness caused by contamination of the working fluid, which leads to malfunctioning of the units.

According to the results of research [14], it has been established that among the main elements of side-loading garbage trucks, the hydraulic system has the shortest service life. It is the key factor in the intensive wear and tear of these vehicles and determines their operational reliability. An analysis of the causes of failures, presented in [15], has made it possible to develop a structural overview of the most common malfunctions of hydraulic equipment in garbage trucks. In particular, the largest percentage is accounted for by failures of hydraulic cylinders – 34.92%, caused by wear of seals, gaskets, rods, destruction of the piston mounting nut, bending of the rod, or mechanical damage. Hydraulic pump failures account for 16.40% and are most often associated with body wear, gear wear, seal damage, or cracks in the body. Next in order of frequency are pipes and hoses – 15.34%, whose failures are caused by breakage and wear. A significant proportion is also accounted for by malfunctions of hydraulic distributors – 13.23%, associated with wear of seals and spools or the formation of cracks in the housing.

A scientific article [16] has established that "conical" wear of the hydraulic cylinder rod within the range of 0.2–0.4 mm along its length before the first major overhaul causes a 7.2% decrease in pressure in the hydraulic system, an increase in specific fuel consumption by 11.4%, and an increase in the carbon monoxide content in exhaust gases by 26%. A further increase in rod wear in the working area to 0.6–0.7 mm leads to a 13.4% drop in pressure, a 21.3% increase in specific fuel consumption, and a sharp increase in the toxicity of exhaust gases—from 25% to 59%, which significantly exceeds the permissible norms. The maximum permissible value of wear of the geometric parameters of the hydraulic cylinder rod of the hydraulic drive of construction and road machines is proposed to be considered to be no more than 0.4 mm. In addition, it has been established that rod wear negatively affects the physical and chemical characteristics of the working fluid: the content of iron and mechanical impurities in it doubles. In turn, it requires more frequent fluid replacement and leads to its overconsumption, which significantly reduces the efficiency and durability of the hydraulic drive, shortening the service life of the machines.

The materials of work [17] point out that the wear of sealing elements in hydraulic systems causes the gradual penetration of working fluid into the non-working cavities of hydraulic machines. Although this process often has no external manifestations, it causes unproductive losses of hydraulic drive power, which, in turn, leads to increased consumption of fuel and lubricants and reduced energy efficiency of the working mechanisms. Power losses associated with seal wear can cause deviations in the operating modes of the hydraulic motor from the optimal ones, which negatively affects the overall performance of the hydraulic drive. The study examines the mechanical system "hydraulic cylinder – sealed piston – compressed hydraulic fluid" and establishes the dependence of the hydraulic cylinder's efficiency on the amount of leakage. Additionally, the process of piston settling during the use of VMGZ working fluid was analyzed and the mechanism of fluid leakage through the hydraulic cylinder seal was investigated.

A scientific article [18] notes that the results of long-term observations of garbage trucks show that the largest percentage of failures is associated with wear and corrosion of the working surfaces of their working equipment parts. In particular, failures of hydraulic cylinders caused by wear of mating surfaces and deformation of the rod and cylinder during operation account for about 32% of the total number of hydraulic drive component failures. Such a high failure rate is explained by uneven loading of the body and intense abrasive wear, which is characteristic of the difficult operating conditions of garbage trucks. An analysis of the causes of failures showed that the key factor in degradation is the wear of the working surfaces of the main parts of the hydraulic drive, in particular the spools and housings of the hydraulic distributors, as well as the rods of the hydraulic cylinders. It has been established that the determining mechanism of wear is hydroabrasive damage, which develops as a result of untimely replacement of the working hydraulic fluid and the use of low-quality or already partially destroyed sealing elements, primarily hydraulic cylinder seals. Dust, mechanical impurities, and wear products enter the sliding zone under such conditions, which significantly accelerates the destruction of working surfaces. The authors propose the usage of chrome plating in a cold self-regulating electrolyte as one of the most effective methods for restoring worn parts. This technology provides the formation of high-quality chrome coatings with increased wear resistance and high process productivity, which makes it a promising alternative to traditional repair methods.

An analytical study of the mathematical model of the process of grinding polymer waste in the grinding chamber of a rotary crusher with continuous classification of the finished product, carried out in the work [19], which provide a detailed understanding of the grinding process based on a comprehensive description of the interaction of particles with the working elements of the crusher. The developed model covers both the dynamics of particle motion in the grinding chamber and the processes of destruction of polymer raw materials under the action of impact, compressive, and shear loads, which makes it possible to accurately predict the parameters of the input material. The authors of the study found that the proposed approach ensures high accuracy in determining the geometric dimensions of the final product particles by taking into account a complex of factors: the angular velocity of the rotor, the initial dimensions of the waste, the geometry and design characteristics of the crusher's working parts, the intensity of raw material feed, and the loading modes of the grinding chamber. In addition, the obtained model made it possible to determine the productivity of the equipment depending on the parameters of the working process, to establish the influence of the classification speed of the finished product on the stability of the crusher, and to estimate the energy costs required to achieve the specified grinding quality. Thanks to the combination of analytical methods and consideration of a wide range of technological parameters, the results of the study can be used to optimize the design of rotary crushers, increase their energy efficiency, and ensure the stability of the particle size distribution of the final product during the processing of polymer waste.

The article [20] presents an algorithm for numerical and analytical research of dynamic processes of a planar six-bar linkage mechanism of a sewing machine's thread puller. The proposed approach involves solving the differential equation of motion of the mechanism using numerical methods, which provides calculation of the kinematic and dynamic characteristics of the system. To verify the accuracy of the obtained results and to visualize the behavior of the mechanism, computer modeling was created in Mathcad software, which made it possible to analyze the trajectories of the links, the velocities and accelerations of the joints, and to identify possible critical points in the mechanism's operation. Such a complex approach ensures both the accuracy of calculations and the possibility of their usage in the optimization of the thread puller design to increase the reliability and productivity of sewing equipment.

In the work [21], an analytical study of a mathematical model was carried out, which made it possible to identify the main dependencies of operation of vibration and vibration-impact machines that use a hydraulic pulse drive with a single-stage pulsator valve. The study covers the analysis of the dynamic characteristics of such machines, in particular the frequency of oscillations, vibration amplitudes, and the periodicity of pulses generated by the pulsator valve. The results obtained allow to estimate the influence of the drive design parameters on the efficiency of the equipment and can be used to optimize its operating modes in order to increase productivity, reduce energy consumption, and reduce wear on working parts.

The article [22] proposes an improved nonlinear mathematical model of the hydraulic drive of a garbage truck's sealing plate mechanism, which takes into account the effect of wear on its hydraulic cylinder. The usage of this model made it possible to conduct a numerical study of the dynamics of the drive, which allowed evaluating the change in the main characteristics of the system during operation. In particular, it was found that taking into account the wear of the hydraulic cylinder significantly affects such parameters as the speed and force of the plate movement, the working pressure in the pressure line, and the stability of the hydraulic fluid supply. It indicates the critical need to take into account operational wear when designing and optimizing hydraulic drives of garbage trucks to ensure their reliable and efficient operation for the entire service life.

The work [23] studies the dynamics of the hydraulic drive of a MSW pressing plate based on a nonlinear mathematical model described by a system of differential equations with corresponding boundary conditions. This model simulates the operation of the hydraulic drive of the garbage truck's pressing sealing plate mechanism, including for static waste compaction modes. In addition, the study proposes and analytically validates a linearized mathematical model of the plate's hydraulic drive, which provides a simplified analysis of the control processes and dynamic behavior of the system. At the same time, these mathematical models do not take into account the wear of the power hydraulic cylinder, one of the key elements of the hydraulic drive, which carries a significant part of the load during the pressing process. Ignoring this factor may limit the practical value of the model, especially regarding long-term prediction of the efficiency and stability of the mechanism in real operating conditions. Since the wear of the hydraulic cylinder significantly affects the performance and reliability of the system, its elimination may lead to appear the deviations between the calculated and actual performance parameters of the hydraulic drive.

However, during the analysis of known publications, the authors did not find a linearized improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder and the results of its analytical study.

Aims of the article

Analytical study of a linearized improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder, to obtain analytical dependencies of the main power and kinematic characteristics of this hydraulic drive in steady-state operation, which can be used during design calculations of new garbage truck designs, taking into account the wear of the working bodies.

Methods

During the analytical study of the linearized improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder, the following methods were used: operator calculus using Laplace transformation to solve a system of ordinary linear differential equations, linearization of nonlinear dependencies, decomposition of complex expressions into simpler fractions, as well as computer modeling methods.

To plot graphs, the MatModel software was used, which implements the 4th order Runge-Kutta-Felberg numerical method with a variable integration step.

Results

Fig. 1 shows the calculation diagram of a linearized improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder when using the rear loading scheme for solid waste during static compaction. The diagram shows the following structural elements: PP – pressing plate; HC – hydraulic cylinder; HD – hydraulic distributor; P – hydraulic pump; SV – safety valve; F – filter; T – working fluid tank, as well as the main geometric, kinematic and power parameters: p_1, p_2, p_3, p_4 – pressures at the pump outlet, hydraulic cylinder inlet, hydraulic cylinder outlet, and filter inlet, respectively; W_1, W_2, W_3, W_4 – volumes of pipelines between the pump and hydraulic distributor, hydraulic distributor and hydraulic cylinder inlet, hydraulic cylinder outlet and hydraulic distributor, hydraulic distributor and filter; Q_P – actual pump flow rate; S_P – cross-sectional area of the distributor opening; S_f – surface area of the filter element; k_f – specific filter capacity (not shown in the diagram); μ_d – dynamic viscosity coefficient (not shown in the diagram); D, d – diameters of the piston and rod; G_p – weight of the pressing plate; G_c – weight of the hydraulic cylinder; G_{W1} – weight of the waste above the pressing plate; G_{W2} – weight of the waste outside the pressing plate; F_{FR} – friction force between the pressing plate and the guides; F_{TW} – the friction force between the MSW and the body; F_C – the force developed by the hydraulic cylinder; h_1, h_2 – the heights of the bottom and top of the press plate; b – the width of the press sealing plate (not shown in the diagram); δ – the thickness of the press plate; α – the angle of inclination of the press plate; x – the displacement of the press plate.

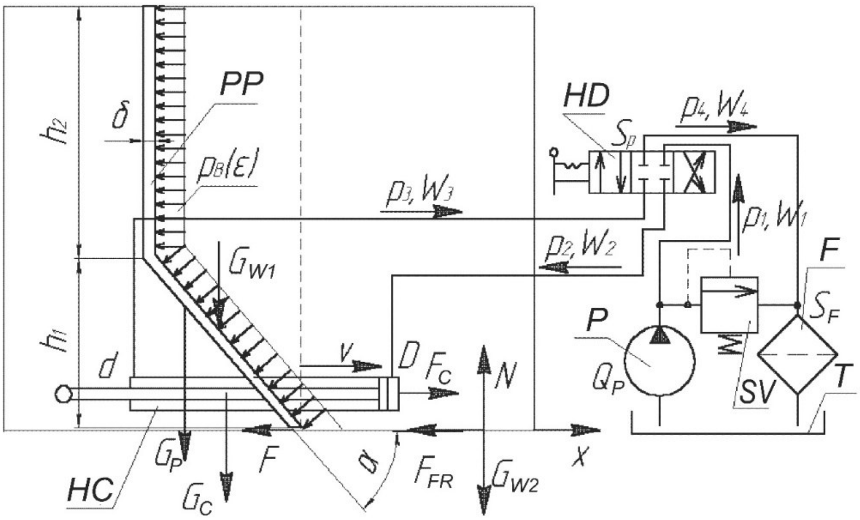

Fig. 1. Calculation scheme of the linearized improved mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder

Analysis of the carried studies of the nonlinear mathematical model [22] showed that $p_1 \approx p_2 \approx p_{12}$, and the influence of pressure in the drain lines and viscous friction forces on the operation of the hydraulic drive is insignificant.

The process of MSW compaction in a garbage truck can be described by the following system of linearized differential equations:

$$\begin{cases} Q_P = vS_{C1} + \sigma_0 p_{12} + \alpha_\sigma + \beta_\sigma t + KW_{12}\dfrac{dp_{12}}{dt}; & (1) \\ p_{12}S_{C1} = m_m \dfrac{dv}{dt} + p_{B0}S_{P1}e^{\beta_p t}, & (2) \end{cases}$$

where $W_{12} = W_1 + W_2$; σ_0 – nominal coefficient of working fluid losses for flow from the high pressure region to the low pressure region, m^5/(N·sec); α_σ, β_σ – approximation coefficients of the dependence of working fluid losses on the duration of the hydraulic cylinder wear process ($\alpha_\sigma = -3.573 \cdot 10^{-9}$ m^3/sec; $\beta_\sigma = 1.443 \cdot 10^{-9}$ m^3/sec^2), m_m – reduced mass of moving parts, kg; v – speed of movement of the garbage truck's sealing plate, m/sec; p_{V0} – initial pressure of MSW compaction, Pa; β_p – approximation coefficient of dependence of MSW compaction pressure on the duration of the process ($\beta_p = 0.00217$).

For further study of the linearized mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder, we will use the Laplace transformation, according to which we obtain the following:

$$\begin{cases} \dfrac{Q_P}{s} = V(s)S_{C1} + P(s)\sigma_0 + \dfrac{\alpha_\sigma}{s} + \dfrac{\beta_\sigma}{s^2} + P_{12}(s)sKW_{12}; & (3) \\ P_{12}(s)S_{C1} = V(s)sm_m + \dfrac{p_{B0}S_{P1}}{s - \beta_p}. & (4) \end{cases}$$

Substituting equation (3) into equation (4), we obtain the following:

$$P_{12}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s(s - \beta_p)(a_2 s^2 + a_1 s + a_0)}, \tag{5}$$

where $a_2 = KW_{12}m_p$; $a_1 = \sigma_0 m_m$; $a_0 = S_{C1}^2$; $b_1 = p_{B0}S_{P1}S_{C1} - \beta_m m_m (Q_P - \alpha_\sigma) - \beta_\sigma m_m$; $b_0 = \beta_\sigma \beta_m m_m$; $b_2 = (Q_P - \alpha_\sigma)m_m$. (6)

By decomposing expression (5) into simpler fractions after reducing it to canonical form, we obtain:

$$P_{12}(s) = A\frac{1}{s} + B\frac{1}{s-\beta_p} + \frac{C}{a_2}\frac{s + a_1/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0a_2 - a_1^2)/(4a_2^2)} + \tag{7}$$
$$+\frac{2D - Ca_1/a_2}{\sqrt{4a_0a_2 - a_1^2}}\frac{\sqrt{4a_0a_2 - a_1^2}/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0a_2 - a_1^2)/(4a_2^2)},$$

where $A = -\frac{b_0}{\beta_p a_0}$; $B = \frac{\beta_p^2 b_2 + b_0}{\beta_p(\beta_p^2 a_1 + \beta_p a_1 + a_0)}$; $C = \frac{a_2}{\beta_p}\left(\frac{b_0}{a_0} - \frac{\beta_p^2 b_2 + b_0}{\beta_p^2 a_1 + \beta_p a_1 + a_0}\right)$;

$$D = -\frac{b_1}{\beta_p} - \frac{b_0(a_0 - \beta_p a_1)}{\beta_p^2 a_0} + \frac{a_0(\beta_p^2 b_2 + b_0)}{\beta_p^2(\beta_p^2 a_1 + \beta_p a_1 + a_0)}. \tag{8}$$

Let's find the original image of (7):

$$p_{12}(t) = A + Be^{\beta_p t} + B_1\cos(\omega_0 t) + \frac{C}{a_2}e^{-\frac{a_1}{2a_2}t}\cos\left(\frac{\sqrt{4a_0a_2 - a_1^2}}{2a_2}t\right) + \frac{2D - Ca_1/a_2}{\sqrt{4a_0a_2 - a_1^2}}e^{-\frac{a_1}{2a_2}t}\sin\left(\frac{\sqrt{4a_0a_2 - a_1^2}}{2a_2}t\right). \tag{9}$$

Excluding insignificant coefficients of expression (9), which have a higher order of smallness, and taking into account the accepted designations according to (6), (8), the pressure in the pressure cavity of the hydraulic cylinder of the garbage truck's sealing plate mechanism, taking into account its wear, is described by the equation:

$$p_{12}(t) \approx \frac{m_m\left[(Q_P - \alpha_\sigma)\beta_m e^{\beta_p t} - \beta_\sigma\right]}{S_{C1}^2}. \tag{10}$$

Solving the system of equations (3, 4) with respect to $V(s) = X(s)s$, after reduction to the canonical form we obtain the following:

$$X(s) = \frac{V(s)}{s} = \frac{B\sigma_0 - C_x\beta_m}{S_{C1}\beta_p}\frac{1}{s} + \frac{Q_P - \alpha_\sigma}{S_{C1}}\frac{1}{s^2} - \frac{\beta_\sigma}{2S_{C1}}\frac{1}{s^3} - \frac{B(KW_{12} + \sigma_0/\beta_m)}{S_{C1}}\frac{1}{s-\beta_m} - \tag{11}$$
$$-\frac{D_x}{a_2S_{C1}}\frac{s + a_1/(2a_2)}{\left[s + a_1/(2a_2)\right]^2 + (4a_0a_2 - a_1^2)/(4a_2^2)} - \frac{2E_x - D_xa_1/a_2}{S_{C1}\sqrt{4a_0a_2 - a_1^2}}\frac{\sqrt{4a_0a_2 - a_1^2}/(2a_2)}{\left[s + a_1/(2a_2)\right]^2 + (4a_0a_2 - a_1^2)/(4a_2^2)},$$

where $A_x = -\sigma_0/\beta_m$; $B_x = KW_{12} + \sigma_0/\beta_m$; $C_x = D\sigma_0/a_0$; $D_x = CKW_{12} - a_2D\sigma_0/a_0$;

$$E_{1p} = C\sigma_0 + DKW_{12} - a_1D\sigma_0/a_0. \tag{12}$$

Next, we find the original image of (11)

$$x(t) = \frac{B\sigma_0 - C_x\beta_p}{S_{C1}\beta_p} + \frac{Q_H - \alpha_\sigma}{S_{C1}}t - \frac{\beta_\sigma}{2S_{C1}}t^2 - \frac{B(KW_{12} + \sigma_0/\beta_m)}{S_{C1}}e^{\beta_p t} + \tag{13}$$
$$-\frac{D_x}{a_2S_{C1}}e^{-\frac{a_1}{2a_2}t}\cos\left(\frac{\sqrt{4a_0a_2 - a_1^2}}{2a_2}t\right) - \frac{2E_x - D_xa_1/a_2}{S_{C1}\sqrt{4a_0a_2 - a_1^2}}e^{-\frac{a_1}{2a_2}t}\sin\left(\frac{\sqrt{4a_0a_2 - a_1^2}}{2a_2}t\right).$$

Neglecting the insignificant coefficients of equation (13), which have a higher order of smallness, and taking into account the accepted designations according to (6), (8), (12) and the initial conditions $x(0) = 0$, the movement of the garbage truck's sealing plate, taking into account the wear of the hydraulic cylinder, can be described by the equation

$$x(t) \approx \frac{Q_P - \alpha_\sigma}{S_{C1}}t - \frac{\beta_\sigma}{2S_{C1}}t^2 + \frac{m_p(Q_P - \alpha_\sigma)(KW_{12} + \sigma_0)}{S_{C1}^3}\left(1 - e^{\beta_p t}\right). \tag{14}$$

After differentiating the equation (14) and taking into account the initial conditions $v(0) = 0$, the speed of movement of the garbage truck's sealing plate, taking into account the wear of the hydraulic cylinder, can be described by the following equation

$$v(t) \approx \frac{m_m \beta_m \left(Q_P - \alpha_\sigma\right)\left(KW_{12} + \sigma_0\right)}{S_{C1}^3}\left(1 - e^{\beta_m t}\right) - \frac{\beta_\sigma}{S_{C1}} t. \tag{15}$$

From equation (10), we can determine the duration of solid waste compaction in a garbage truck, taking into account the wear of the hydraulic cylinder of the sealing plate.

$$t \approx \frac{1}{\beta_m} \ln\left[\frac{p_{12} S_{C1}^2}{m_m \beta_m \left(Q_P - \alpha_\sigma\right)} + \frac{\beta_\sigma}{\beta_m \left(Q_P - \alpha_\sigma\right)}\right]. \tag{16}$$

A comparison of the results that were obtained using nonlinear and linearized mathematical models of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder, as well as using equations obtained as a result of the analytical solution of the linearized model, is shown in Fig. 2.

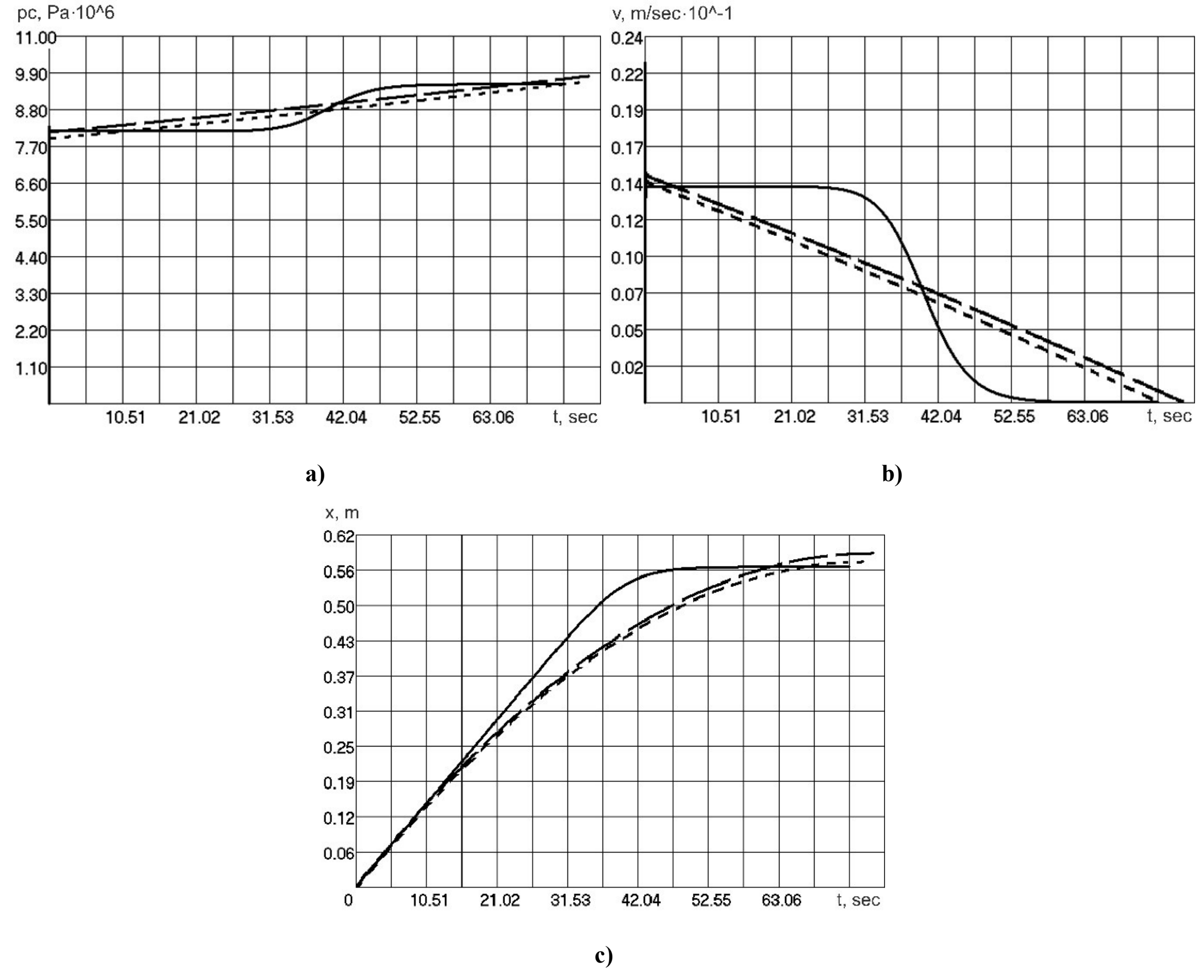

Fig. 2. Comparison of results obtained using nonlinear (——) and linearized (– – –) mathematical models of the hydraulic drive of the garbage truck's sealing plate mechanism taking into account the wear of its hydraulic cylinder, as well as using the equations obtained as a result of its analytical solution (···): a) –change in pressure in the hydraulic cylinder; b) –change in speed; c) –displacement

When comparing the characteristics of MSW compaction in a garbage truck obtained using a nonlinear mathematical model and equations (10), (14), (15), derived from the analytical solution of the linearized mathematical model of the hydraulic drive of the garbage truck's sealing plate mechanism, taking into account the wear of its hydraulic cylinder, the deviation is about 10% compared to the nonlinear mathematical model, which is acceptable for preliminary design calculations. If necessary, the values of the main parameters can be refined at the final stage of design using a nonlinear mathematical model.

The obtained regression equation (16) allows to approximately determine the duration of MSW compaction in a garbage truck, taking into account the wear of the hydraulic cylinder of the sealing plate, which can be used during design calculations of new garbage truck constructions, taking into account the wear of the working bodies without the need to research the nonlinear mathematical model of the drive of its working bodies, as well as during the optimization of the main parameters of the hydraulic drive.

The development of an improved methodology for design calculations of new garbage truck structures, taking into account the wear and tear of operational mechanisms, requires additional scientific research and further development of the relevant theoretical foundations.

Conclusions

To perform design calculations for new modifications of garbage trucks, approximate analytical equations were obtained that describe the change in pressure over time in the pressure line of the hydraulic cylinder, as well as the speed and displacement of the sealing plate based on the proposed linearized mathematical model of the hydraulic drive of the garbage truck's compaction sealing plate mechanism, taking into account the wear of the hydraulic cylinder. The resulting regression equation provides the ability to approximately determine the duration of the solid waste compaction process in a garbage truck, taking into account the wear of the hydraulic cylinder. This equation can be used when designing new garbage truck constructions, allowing for changes in the technical condition of the operational mechanisms without the need to analyze a complete nonlinear mathematical model of the drive of the working bodies. It has been established that the development of an improved methodology for design calculations of new garbage truck constructions, taking into account the wear of the operational mechanisms, requires additional scientific research and further development of the relevant theoretical foundations.

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Березюк О.В., Савуляк В.І., Харжевський В.О., Ivanov S.Cv., Алексеєв А.Є. Аналітичне дослідження удосконаленої математичної моделі гідроприводу механізму ущільнюючої плити сміттєвоза із урахуванням зносу його гідроциліндра.

Стаття присвячена аналітичному дослідженню вдосконаленої математичної моделі гідроприводу механізму ущільнювальної плити сміттєвоза, у якій додатково враховано вплив зношення гідроциліндра на його роботу. На основі проведених числових досліджень удосконаленої нелінійної математичної моделі гідроприводу механізму ущільнювальної плити сміттєвоза, що враховує зношування гідроциліндра, було сформовано її лінеаризовану модифікацію, подану у вигляді системи звичайних лінійних диференціальних рівнянь другого порядку. Для виконання проектних розрахунків нових модифікацій сміттєвозів отримано наближені аналітичні вирази, що описують зміну в часі тиску в напірній магістралі гідроциліндра, а також швидкість і переміщення ущільнювальної плити на основі запропонованої лінеаризованої математичної моделі гідроприводу механізму ущільнюючої плити сміттєвоза з урахуванням зношування гідроциліндра. Отримане рівняння регресії, що забезпечує можливість орієнтовного визначення тривалості процесу ущільнення твердих побутових відходів у сміттєвозі з урахуванням зносу гідроциліндра. Це рівняння може бути застосоване під час проектування нових конструкцій сміттєвозів, дозволяючи враховувати зміну технічного стану виконавчих механізмів без необхідності проведення аналізу повної нелінійної математичної моделі приводу робочих органів. Крім того, воно є придатним для використання в процедурах оптимізації основних параметрів гідроприводу. З’ясовано, що формування удосконаленої методики проектних розрахунків нових конструкцій сміттєвозів з урахуванням зношування виконавчих механізмів потребує проведення додаткових наукових досліджень та подальшого розвитку відповідного теоретичного апарату.

Ключові слова: лінеаризована математична модель, урахування зносу, перетворення за Лапласом, гідропривод, знос, гідроциліндр, механізм ущільнюючої плити, сміттєвоз, тверді побутові відходи