



Technological features of forming regular micro reliefs on conical surfaces

V.O. Dzyura, *[0000-0002-1801-2419](https://orcid.org/0000-0002-1801-2419), S.S. Kyryk [0009-0002-6649-7649](https://orcid.org/0009-0002-6649-7649)

Ternopil Ivan Puluy National Technical University, Ukraine

*E-mail: volodymyrdzyura@gmail.com

Received: 28 March 2026; Revised 25 April 2026; Accept: 04 May 2026

Abstract

The paper examines the technological features of schemes for forming regular micro reliefs on conical surfaces. A classification of these formation schemes is proposed according to the regularity of the generated micro relief and its geometric parameters. A set of possible groove configurations of the formed micro relief has been obtained, comprising 54 variants of geometric property combinations that integrate the shape of the axial line of the micro relief grooves, the pattern of variation of the groove geometric parameters, and the groove profile itself. The specific characteristics of each technological scheme are analyzed, and the principal analytical relationships are derived that determine the interdependence between the geometric parameters defining the shape of the micro relief groove elements—namely the pitch (T_k) and the amplitude (A_g); the groove arrangement parameter – the center-to-center spacing (S_o); and the parameters of the conical surface, including the cone angle (α_c) and the length of the axial line of the regular micro irregularities formed on the conical surface. The obtained relationships make it possible to ensure the regularity of the micro relief groove elements and to form a micro relief with the required geometric parameters. It has been established that the determination of the length of the helical line on the conical surface forms the basis for further calculations of the groove elements arranged along this line. An analytical expression for determining the length of this line has been obtained, which serves as a foundation for ensuring the regularity of micro relief grooves formed along a helical line according to the method of geometric similarity.

Keywords: regular micro reliefs, grooves, conical surfaces, geometric parameters.

Introduction

Modern mechanical engineering increasingly relies on the concept of the "functional surface," where the condition of the subsurface layer determines a component's service life, energy efficiency, and reliability as significantly as material strength or geometric precision. This is particularly critical for machine parts designed as "bodies of revolution," as they operate in high-precision friction pairs involving rolling or sliding contact: shafts and bushings, bearing journals, rollers, liners, plunger and spool pairs, sealing elements, and the working surfaces of vibration and pumping units. In such systems, contact pressures, cyclic loads, local temperature spikes, and complex lubrication regimes—ranging from hydrodynamic to boundary lubrication—create stress concentration zones and potential reductions in durability. Consequently, the controlled formation of micro-reliefs to enhance surface performance characteristics remains a vital challenge for contemporary mechanical engineering.

Literature review

Surface engineering involves a significant volume of scientific research aimed at developing innovative technological solutions in mechanical engineering to meet the growing demands of the industry. The use of traditional surface treatment methods is approaching its technical limits, restricting the possibilities for enhancing and imparting the necessary performance characteristics to the surfaces of machine parts.

Modern mechanical engineering actively employs methods such as surface plastic deformation, chemical, and thermochemical treatment to improve the functional properties of surfaces. These methods are well-studied and widely applied to ensure the specified characteristics of the working surfaces of components [1, 2, 3]. They guarantee the stability of the physicochemical, mechanical, and operational properties of the formed surfaces throughout their entire service life.



The feasibility of forming regular micro-reliefs on the functional surfaces of machine parts is primarily driven by the ability to purposefully control the functional properties of the surface layer at the micro-geometric level. Unlike stochastic roughness, a regular texture ensures a predictable contact nature, stable lubrication conditions, and a controlled load distribution within the friction zone. This facilitates a transition from empirical parameter selection to the engineering design of surfaces with predetermined tribological, hydrodynamic, and wear-resistant characteristics. Research in the field of surface plastic deformation has demonstrated that ordered micro-grooves or cellular structures act as lubricant micro-reservoirs, reduce the real contact area, and promote the formation of a stable lubricant film, leading to a reduction in the friction coefficient and contact zone temperature [4]. Generally, it can be asserted that textured surfaces produced via burnishing and rolling exhibit enhanced wear resistance and contact endurance due to the strengthening of the surface layer and the formation of a favorable micro-topography. For surfaces with regular micro-reliefs, one of the most critical parameters is the relative area of the micro-relief—a value indicating the ratio of the projected area of the micro-relief grooves to the total surface area on which they are located [5]. The formation of regular micro-reliefs on flat surfaces does not present significant difficulties and is easily implemented at the current level of technological development. On flat surfaces, it is possible to realize any micro-relief shape and ensure complete regularity. Regularity is defined as the stability of the micro-relief's geometric parameters in two mutually perpendicular directions. Micro-reliefs on flat surfaces can be formed on standard milling or drilling machines without the use of specialized devices [6].

A more complex case involves the formation of regular micro-reliefs on internal or external cylindrical surfaces, particularly stepped ones. In this instance, the profile of the micro-relief grooves is somewhat distorted, with the magnitude of this distortion depending on the curvature of the surface being textured. Even more challenging is the formation of regular micro-reliefs on the face surfaces of bodies of revolution. Such surfaces are found on variator transmission discs, rod ends, face flanges, and other machine components. Analytical dependencies for determining the geometric parameters of triangular-profile micro-relief grooves are defined in [7]. A specific feature of forming micro-relief grooves on the face surfaces of bodies of revolution is that the linear velocity of a point decreases as it approaches the center of rotation, necessitating constant correction of the processing regimes. Furthermore, it is impossible to form a completely regular micro-relief on such a surface because the circumference decreases toward the center of rotation. Consequently, grooves located on this inner circumference will have a smaller circular pitch than those located on a circumference with a larger radius. The most complex case involves intricate profiled surfaces: involute, spherical, conical, and other complex geometries. The difficulty lies in the fact that the geometric parameters of the grooves (amplitude, axial pitch) change continuously. Consequently, the relative area of the micro-relief will vary across different sections, leading to discrepancies in the surface's operational properties. However, the application of three- or five-axis CNC machine tools enables the reproduction of grooves in practically any configuration, while stepless feed regulation guarantees high geometric precision and texture parameter stability. This results in a fully regular structure with cells of the required, even highly complex, shape.

Typically, the tool used for forming micro-relief grooves allows for the regulation of the deforming force, utilizing a ball as the deforming element. The proposed tool facilitates the generation of various texture types with grooves of arbitrary configurations, where the design process involves preliminary mathematical modeling in the MathCAD environment. Such modeling determines the coordinates of characteristic profile points used to prepare the CNC control program. This allows for the pre-standardization of micro-relief geometric parameters, evaluation of its relative area, and prediction of the textured surface's operational properties. The advantages of this approach include versatility regarding groove shape and size, as well as the ability to purposefully form the required physical and mechanical characteristics of the surface layer. The primary limitations remain the high cost of equipment and tooling, alongside relatively low productivity.

Another approach is based on the copying or rolling method [8], where the decisive role is played by a tool with forming elements whose geometry directly corresponds to the configuration of the intended grooves. In this case, the surface structure is reproduced through the mechanical transfer of the tool profile onto the workpiece material, where the relationship between the feed rate and the rotational speed of the working unit determines the spatial arrangement of the grooves and the shape of their lateral flanks.

The most effective technical solutions for forming various types of regular micro-reliefs involve the use of tools and equipment described in [9]. Experimental studies demonstrate that such technological systems enable the formation of regular micro-reliefs of varying complexity on virtually any surface. Software-controlled processing ensures high geometric precision of the formed grooves and, consequently, maintains a stable relative area of the micro-relief.

In study [10], the influence of the arrangement of triangular regular micro-irregularities on the friction coefficient between the end surfaces of bodies of revolution was investigated, both with and without the use of L-AN-46 lubricant. The experiments were conducted with a clamping force of 20 N and a relative sliding velocity of 0.4 m/s. The authors found that the arrangement of micro-relief elements significantly affects the friction coefficient values. The lowest friction coefficient was observed during the interaction of end surfaces with a micro-relief featuring a central angle of 90° oriented toward the outer edges of the end surface.

The mechanisms of regular micro-relief formation and their positive impact on enhancing performance properties are also detailed in [11].

The aim of this work is to derive fundamental analytical dependencies that describe the regularities of regular micro-relief distribution on the conical surfaces of bodies of revolution.

Research materials and methodology

For bodies of revolution, the requirement for regularity takes on additional significance due to the specific kinematics of relief formation. Most technologies used to create micro-reliefs on cylindrical surfaces rely on a combination of workpiece rotation and tool feed, resulting in relief elements formed along helical trajectories or their combinations. Any instability within this kinematic pair – such as runout, synchronization errors between feed and rotation, elastic deformations of the technological system, fluctuations in contact force, or tool wear and micro-slippage – immediately translates into a disruption of the pattern's periodicity. This manifests as variations in pitch and depth, distortion of element geometry, or local rarefaction and oversaturation of the texture. In turn, an irregular micro-relief generates non-homogeneous pressure and temperature fields within the contact zone, impairs the repeatability of the tribological effect, and may facilitate local zones of accelerated wear or the initiation of fatigue damage. Thus, for components designed as bodies of revolution, micro-relief regularity is a fundamental prerequisite for the controllability and functional reliability of the surface. There are three principal schemes for forming regular and partially regular micro-reliefs on conical surfaces (Fig. 1), one of which comprises two subtypes.

The first scheme involves the formation of micro-reliefs with grooves arranged in concentric circles with uniform geometric parameters. In this case, the grooves form a system of closed circles whose centers coincide with the axis of the conical surface. The geometric characteristics of the grooves – width, depth, and the spacing between them – remain constant across all concentric trajectories. This scheme ensures a uniform distribution of micro-asperities across the surface and is utilized when homogeneous tribological properties are required throughout the entire contact area.

The second scheme involves the formation of micro-reliefs with grooves arranged in concentric circles based on the principle of geometric similarity. In this instance, the grooves also form a system of concentric circles; however, their geometric parameters vary proportionally to the distance from the cone's vertex. This approach ensures the scale similarity of micro-relief elements across the entire surface and accounts for the varying local radius of the conical surface.

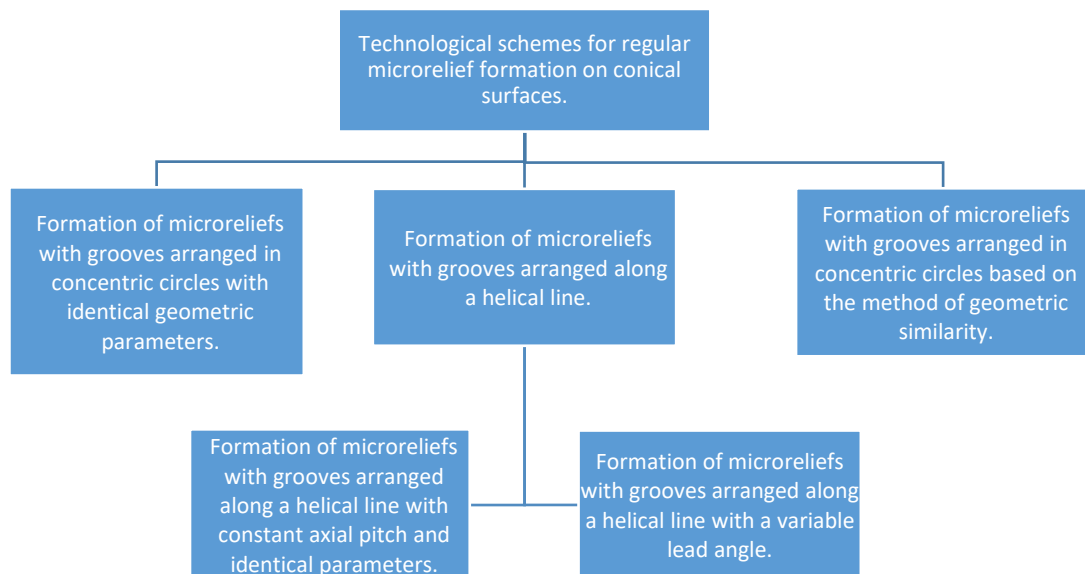


Fig. 1. Classification of technological schemes for regular micro relief formation on conical surfaces

The third scheme involves the formation of micro reliefs with grooves arranged along a helical line. In this case, the groove forms a continuous spiral trajectory on the conical surface. This type of micro relief is generated through a combination of the part's rotational motion and the tool's translational movement. The helical arrangement of the grooves creates a directional surface structure, which can improve lubricant drainage or facilitate the formation of a hydrodynamic lubrication wedge. This scheme is characterized by the formation of grooves along a helical line with a constant axial pitch and groove amplitude. This allows for the adjustment of groove density across different sections of the conical surface, which may be necessary to compensate for changes in contact conditions or loading along the cone.

The fourth scheme shares common features with the second and third schemes; however, the micro relief grooves are formed using the method of geometric similarity. This allows for the maintenance of the ratio between the primary geometric parameters of the groove element, ensuring a proportional change in the shape of the micro

relief grooves as the diameter of the conical surface decreases. These schemes provide for the creation of a set of possible micro relief groove variants formed on conical surfaces (Fig. 2), which includes 54 variants of geometric property sets. These sets combine the shape of the groove centerline, the law governing the change in geometric parameters of the grooves, and the groove shape itself. At the first parameter level, the shape of the groove centerline is defined: it can be in the form of a concentric circle, a helical line with a constant pitch, or a helical line with a constant lead angle. At the second level, the law governing the change in the geometric parameters of the micro relief grooves is specified: constant geometric parameters T_g and A_g allow for the formation of identical micro relief groove elements; parameters determined by the method of geometric similarity in the axial direction ensure a gradual reduction of the groove without losing the ratio between the axial pitch T_g and the groove amplitude A_g ; parameters determined by the method of geometric similarity in the inter-axis direction provide partial regularity of the micro relief with a gradual reduction of the axial pitch.

At the third level, the alignment of the micro relief grooves is established, which is determined by the relationship between the symmetry lines of the groove elements. They can be coaxial, where the symmetry axes of the groove elements located on circles (coils) of different diameters coincide; shifted by $0.5T_k$, which is a condition for creating micro reliefs with grooves whose peaks may touch; and other so-called "creeping" displacements, where the displacement value Δ is within the range $0 < \Delta < 0.5T_k$.

At the final level, the shape of the micro relief groove is selected. The most common are sinusoidal and triangular shapes of continuous micro relief grooves.

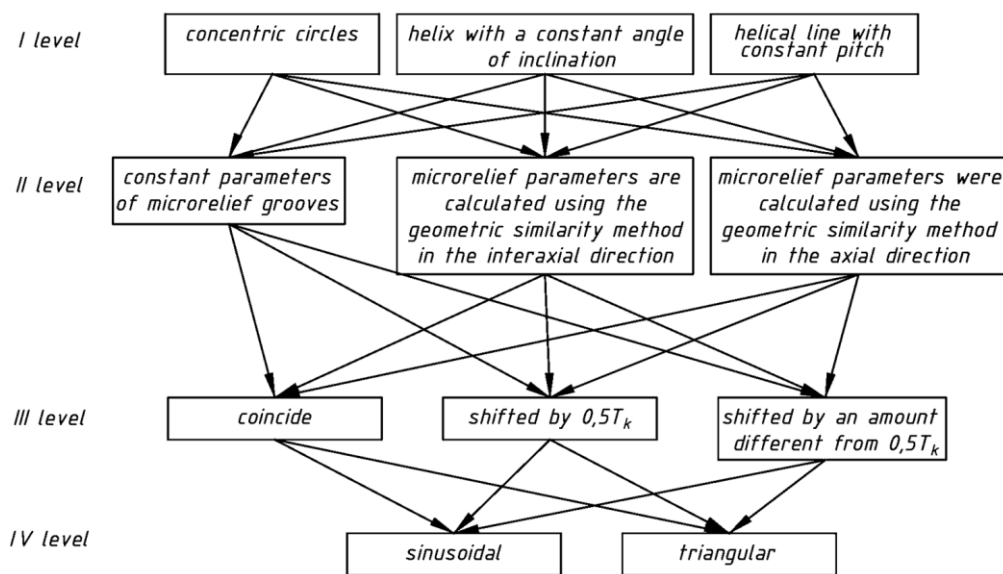


Fig. 2. Set of possible variants for micro relief grooves formed on conical surfaces

The parameterization of the geometric properties of regular micro relief grooves formed on a conical surface is quite complex (Fig. 3); it differs significantly from grooves formed on flat surfaces and more closely resembles grooves formed on the end faces of bodies of revolution [12].

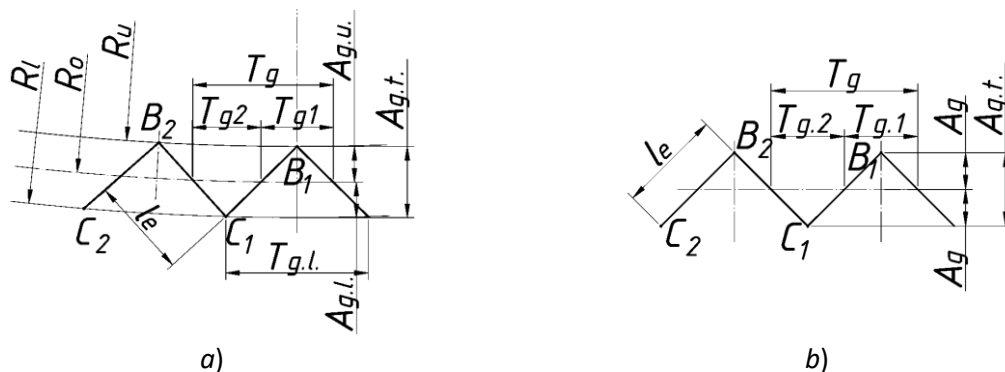


Fig. 3. Parameters of triangular profile microrelief grooves: (a) formed on a conical surface according to the scheme of groove arrangement in concentric circles with identical geometric parameters; (b) formed on a flat surface

As seen in (Fig. 3a), the simplest triangular profile micro relief grooves formed on a conical surface are described by a greater number of geometric parameters due to the asymmetry of the groove profile relative to the longitudinal axis R_o .

The amplitude value $A_{g.u.}$, defined from the groove centerline described by a circle of radius R_o to the peaks lying on a circle of radius R_l , and the amplitude value $A_{g.l.}$, defined from the centerline to the peaks lying on a circle of radius R_u , are not identical, i.e., $A_{g.u.} \neq A_{g.l.}$. Collectively, these values determine the total groove height $A_{g.t.} = A_{g.u.} + A_{g.l.}$. The corresponding parameter for grooves formed on flat surfaces (Fig. 3b) is identical and equals A_g .

The parameter T_g , which describes the axial pitch of the micro relief grooves formed on flat surfaces (Fig. 3b), is defined as the distance between periodically repeating parts of the groove profile and represents the sum of the segments intersecting the groove profile along the centerline, i.e., $T_g = T_{g1} + T_{g2}$.

For grooves formed on conical surfaces (Fig. 3a), the axial pitch T_g is also defined as the distance between periodically repeating parts of the groove profile; however, the segments that constitute it are non-identical, i.e. $T_{g1} \neq T_{g2}$. This occurs because the outer and inner peaks of the micro relief grooves are located on circles of different radii, R_l and R_u . Consequently, the distance between the profile peaks BB_1 will be smaller than the distance between the profile bases CC_1 . Thus, for the analytical description of the geometric parameters of regular micro relief grooves formed on conical surfaces, it is appropriate to employ modified notation for the primary geometric parameters: the axial pitch T_g and the amplitude A_g .

The formation scheme of regular micro reliefs with grooves arranged on concentric circles of a conical surface (Fig. 4) ensures uniform geometric parameters A_g, T_g , which remain constant regardless of the circumference (the centerline of the micro relief grooves) on which the groove elements are located. A distinctive feature of this formation scheme is that the number of groove elements positioned at different levels is non-identical and decreases as they approach the apex of the conical surface. The center-to-center distance S_o between micro relief grooves at adjacent levels is determined by the groove pitch T_g and the inclination angle of the conical surface α_c . The resulting micro relief is partially regular. Regularity is maintained for parameters A_g and T_g , however, given the specific characteristics of the conical surface, the center-to-center pitch S_o of the micro relief grooves is inconsistent. Furthermore, the coaxiality of the micro relief grooves is not maintained. The specific characteristics of forming such micro relief grooves are illustrated in Figure 3.

The method for forming the grooves is implemented as follows. By combining the rotation movement D_n of the workpiece with a conical surface at a rotational speed n_g with the simultaneous reciprocating movement D_i of a ball-shaped deforming element – using an amplitude A_g and an oscillation frequency i_g – grooves are formed with a pitch $T_g = \pi \cdot D_1/m$, where D_1 is the cross-sectional diameter of the surface of the body of revolution and m is the number of groove elements formed per single revolution of the workpiece.

Upon completing one full revolution of the workpiece with the conical surface about its axis, the rotational movement D_n and the oscillatory movement D_i are deactivated. The deforming element is withdrawn from contact with the conical surface and is moved along the workpiece axis by the center-to-center distance S_o using a discrete movement D_s . Subsequently, the deforming element is reintroduced into engagement with the conical surface of the workpiece, the movements D_i and D_n are reactivated, and the formation of the micro relief grooves proceeds.

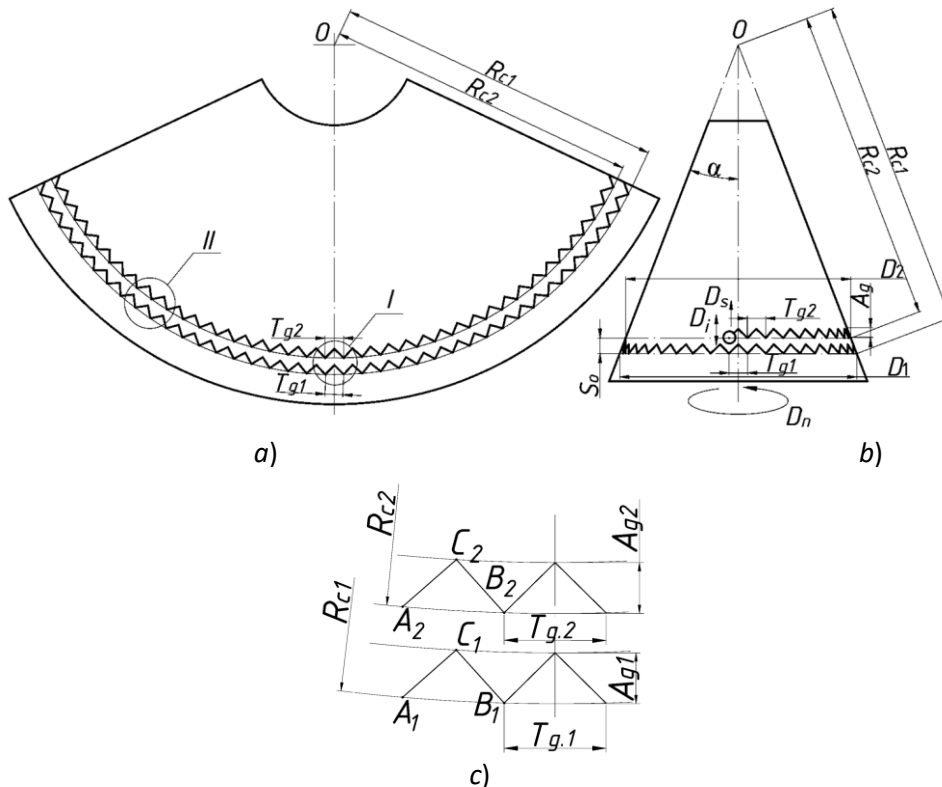


Fig. 4. The formation scheme of a regular micro relief on a conical surface with grooves arranged in concentric circles with identical geometric parameters: (a) development of the conical surface; (b) general view of the conical surface; (c) – parameterization of the micro relief grooves.

The absence of distortion in the profile of a groove formed with uniform geometric parameters is achieved by reducing the number of groove elements on the surface. A disadvantage of this scheme is that, despite ensuring identical primary geometric parameters for the grooves (A_g, T_g), the resulting microrelief will be irregular. This is clearly observed when comparing the coaxiality of grooves located at different levels (Fig. 4a). This occurs because the circular arcs on which the grooves are positioned have different surface curvatures.

The core of the method for forming grooves with uniform geometric parameters involves determining a circle diameter D_2 on the conical surface that can accommodate an integer number of groove elements with the pitch T_g used on a circle of a different diameter D_1 (Fig. 4c). Consequently, the oscillation frequency of the deforming element on circles of different diameters will vary and depends on the number of groove elements.

Analytical dependencies that establish the relationship between the geometric parameters defining the groove shape – specifically the pitch T_k and amplitude A_g – the groove placement parameters (center-to-center distance S_o), and the surface parameters (inclination angle α and the length of the axial line of the regular micro-irregularities) will facilitate ensuring the regularity of the micro relief grooves and the formation of a micro relief with the required specifications.

The length of the arc on which the micro relief groove elements are positioned is determined by the following formula:

$$L_1 = \frac{\pi \cdot R_1 \cdot \alpha}{180^\circ} \quad (1)$$

where R_1 is the radius of a circle with diameter D_1 , mm;
 α is the cone apex angle, deg.

On an arc of radius R_1 , n_1 groove elements are positioned; therefore, the arc length corresponding to a single groove element is defined as:

$$l_{g1} = \frac{\pi \cdot R_1 \cdot \alpha}{n_1 \cdot 180^\circ} \quad (2)$$

Since linear dimensions are used during the design of the grooves, and in order to eliminate the distortion caused by the placement of groove elements along a circular arc, the groove pitch is defined as:

$$T_g = 2 \cdot R_1 \cdot \sin\left(\frac{l_g \cdot 90^\circ}{R_1 \cdot \pi}\right) = 2 \cdot R_1 \cdot \sin\left(\frac{\alpha}{2 \cdot n_1}\right) \quad (3)$$

The radius of the circle on which the groove containing n_2 elements is positioned shall be determined by the following dependency:

$$R_2 = \frac{l_g \cdot n_2 \cdot 180^\circ}{\pi \cdot \alpha} = \frac{R_1 \cdot n_2}{n_1} \quad (4)$$

The center-to-center pitch S_o of the micro relief grooves is determined by the following dependency:

$$S_o = \frac{m \cdot T_g}{2 \cdot \pi \cdot \operatorname{tg} \alpha} \quad (5)$$

where T_g is the axial pitch of the micro relief grooves, mm;
 m – an integer indicating how much smaller the number of micro relief groove elements located on a concentric circle of diameter D_2 is compared to those on a concentric circle of diameter D_1 .

The scheme for forming micro reliefs with grooves arranged in concentric circles using the method of geometric similarity assumes the geometric similarity of the axial pitch T_g , while the groove amplitude A_g remains constant (Fig. 5).

A key feature of this scheme is that the oscillation frequency of the deforming element will remain the same during the formation of micro relief groove elements on each concentric circle of diameter D_2 on the conical surface. Micro relief grooves positioned on concentric circles of different diameters D_1 and D_2 on the conical surface will have an identical amplitude A_g but different axial pitches T_{g2} . The latter is determined via the method of geometric similarity based on the ratio of the diameters of the concentric circles where the micro relief grooves are formed, using the formula: $T_{g2} = T_{g1} \times D_2 / D_1$.

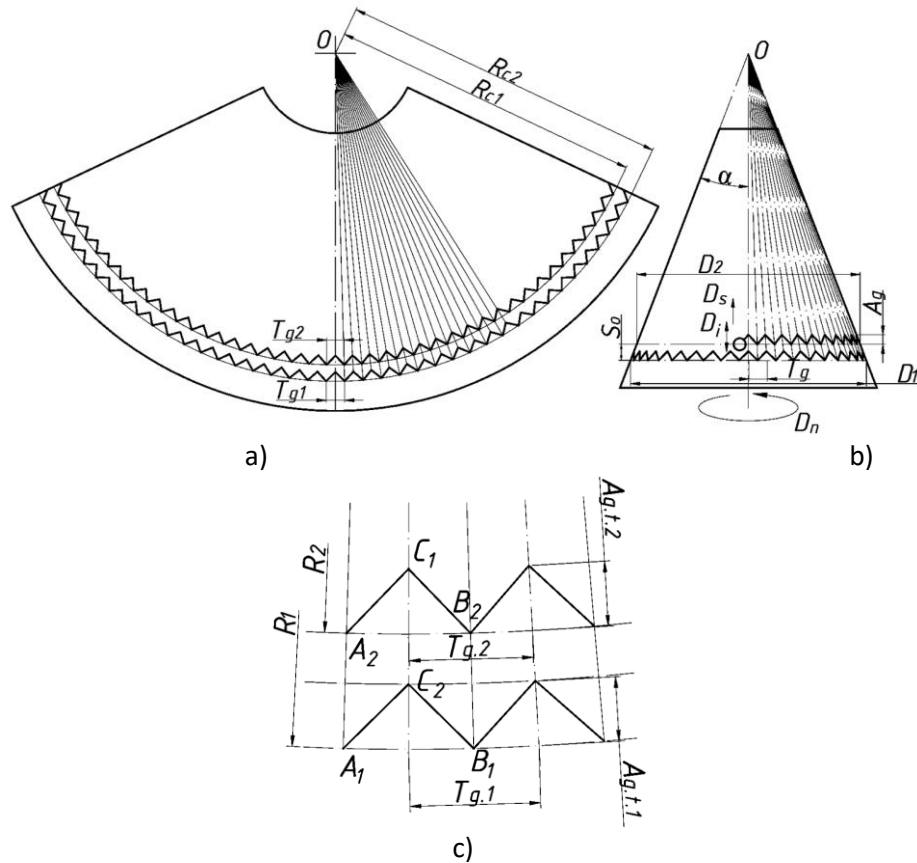


Fig. 5. The formation scheme of a regular microrelief on a conical surface with grooves arranged in concentric circles using the method of geometric similarity (a) development of the conical surface; (b) general view of the conical surface; (c) parameterization of the microrelief grooves.

Technologically, the simplest approach for forming a regular microrelief on a conical surface is a scheme in which the microrelief grooves are arranged along a helical line. This scheme can be implemented in two variants: where the microrelief grooves are positioned along a helical line with a uniform pitch (Fig. 6), and where they are positioned along a helical line with a constant inclination angle but uniform pitch. Considering the technological aspects of implementation, the scheme with a uniform pitch is simpler, as it is ensured by the stability of the machine tool feed rate.

At the same time, this scheme is one of the most complex in terms of determining the geometric parameters of the microrelief grooves, as the grooves are positioned along a helical line described on a conical surface rather than a circular arc (Fig. 6).

The precise determination of the helical line length is a necessary prerequisite for ensuring the regularity of the formed microrelief. A helical line on a conical surface has a complex shape because its radius changes along the axis. Its length is most accurately determined through parametric specification and the integration of an arc element.

The coordinates of a point on a helical line on a cone can be specified as follows:

$$\begin{cases} x = z \cdot \tan(\alpha) \cdot \cos(\varphi) \\ y = z \cdot \tan(\alpha) \cdot \sin(\varphi) \\ z = \frac{S_o \cdot \varphi}{2\pi} \end{cases} \quad (6)$$

where α – angle between the axis and the generator of the conical surface, deg;

φ – angle of rotation around the axis, deg;

S_o – pitch of the helical line (Fig. 6), mm.

From this, the radius on the cone is determined by the formula:

$$r(\varphi) = \frac{S_o}{2\pi} \varphi \cdot \tan(\alpha) \quad (7)$$

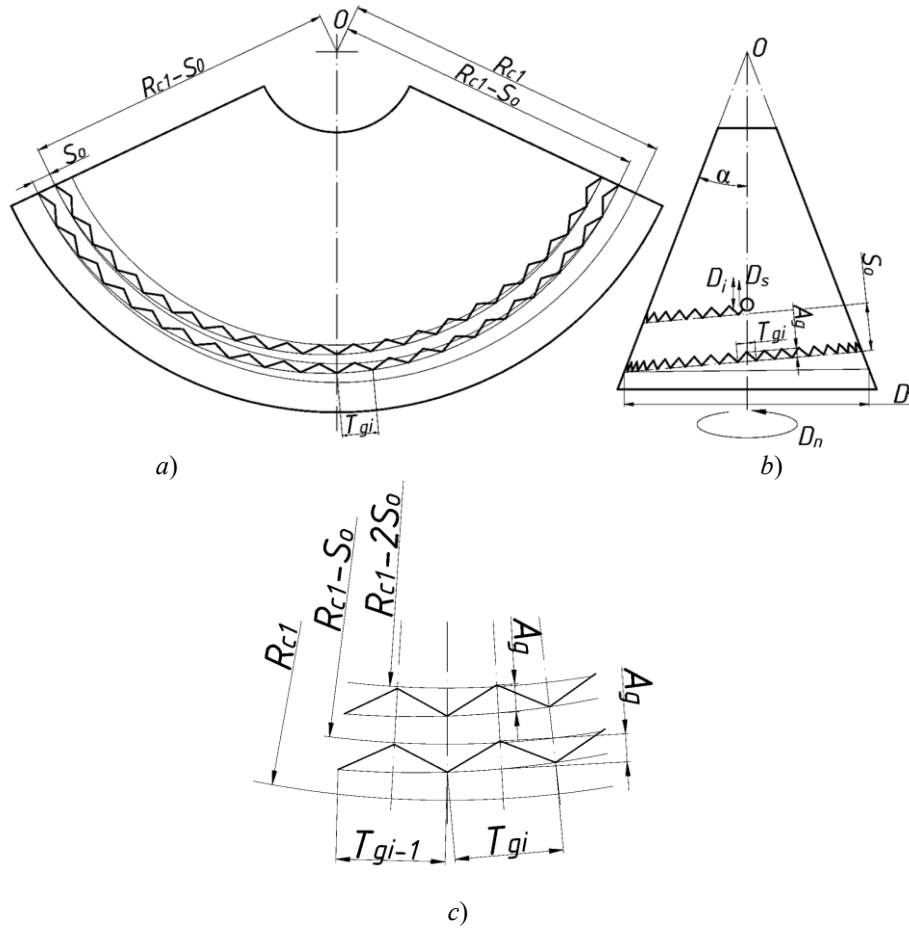


Fig. 6. The formation scheme of a regular micro relief on a conical surface with grooves arranged along a helical line using the method of geometric similarity (a) development of the conical surface; (b) general view of the conical surface; (c) parameterization of the micro relief grooves.

The length of the helical line is determined through the arc element of a spatial curve:

$$ds = \sqrt{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2 + \left(\frac{dz}{d\varphi}\right)^2} d\varphi \quad (8)$$

After substitution, we obtain the formula for the length of the helical line from φ_1 to φ_2

$$L = \int_{\varphi_1}^{\varphi_2} \sqrt{\left(\frac{S_0}{2\pi}\right)^2 + \left(\frac{S_0}{2\pi} \cdot \tan(\alpha)\right)^2 \cdot (1 + \varphi^2)} d\varphi \quad (9)$$

To determine the length of a single turn, the angle values are as follows: $\varphi_1=0$ and $\varphi_2=2\pi$.

However, for practical application, it is more convenient to derive an analytical dependency for determining the helical line length that incorporates the geometric parameters of the conical surface – specifically, the larger R and smaller r radii of the cone.

A precise parametric description of a single turn: if $\varphi \in [0, 2\pi]$, then the radius varies from r to R .

$$\rho(\varphi) = r + \frac{R-r}{2\pi} \varphi \quad (10)$$

Then the spatial curve is defined as:

$$\begin{cases} x = \rho(\varphi) \cdot \cos(\varphi) \\ y = \rho(\varphi) \cdot \sin(\varphi) \\ z = \frac{p \cdot \varphi}{2\pi} \end{cases} \quad (11)$$

where ρ – the current radius of a point on the helical line, i.e., the distance from the cone axis to a point on the curve.

The length of a single turn is determined by the formula:

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2 + \left(\frac{dz}{d\varphi}\right)^2} d\varphi \quad (12)$$

Since

$$\rho'(\varphi) = \frac{R-r}{2\pi} \quad (13)$$

then after differentiation, we obtain the standard result:

$$\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2 = (\rho'(\varphi))^2 + \rho^2(\varphi) \quad (14)$$

Therefore

$$L = \int_0^{2\pi} \sqrt{\left(\frac{R-r}{2\pi}\right)^2 + \left(r + \frac{R-r}{2\pi}\varphi\right)^2 + \left(\frac{S_o}{2\pi}\right)^2} d\varphi \quad (15)$$

For convenience, we introduce the notation

$$a = \frac{R-r}{2\pi}; \quad b = \frac{S_o}{2\pi}; \quad (16)$$

Then

$$L = \int_0^{2\pi} \sqrt{a^2 + b^2 + (r + a\varphi)^2} d\varphi \quad (17)$$

Making a replacement

$$u = r + a\varphi \quad du = ad\varphi; \quad (18)$$

At $\varphi=0$: $u=r$, at $\varphi=2\pi$: $u=R$.

Then

$$L = \frac{1}{a} \int_r^R \sqrt{u^2 + a^2 + b^2} du \quad (19)$$

Let us denote

$$c^2 = a^2 + b^2 \quad (20)$$

That is

$$c^2 = \left(\frac{R-r}{2\pi}\right)^2 + \left(\frac{S_o}{2\pi}\right)^2 = \frac{(R-r)^2 + S_o^2}{4\pi^2} \quad (21)$$

Given the formula for the original indefinite integral

$$\int \sqrt{u^2 + c^2} du = \frac{u}{2} \sqrt{u^2 + c^2} + \frac{c^2}{2} \ln(u + \sqrt{u^2 + c^2}) + C \quad (22)$$

where C – arbitrary constant of integration.

Formula (19) will take the form

$$L = \frac{1}{a} \left[\frac{u}{2} \sqrt{u^2 + c^2} + \frac{c^2}{2} \ln(u + \sqrt{u^2 + c^2}) \right]_r^R \quad (23)$$

Because

$$\frac{1}{a} = \frac{2\pi}{R-r} \quad (24)$$

Then formula (23) will take the form

$$L = \frac{\pi}{R-r} \left[R\sqrt{R^2 + c^2} - r\sqrt{r^2 + c^2} + c^2 \ln \left(\frac{R + \sqrt{R^2 + c^2}}{r + \sqrt{r^2 + c^2}} \right) \right] \quad (25)$$

where

$$c^2 = \frac{(R-r)^2 + S_o^2}{4\pi^2}; \quad (26)$$

Taking into account (26), formula (25) takes the following form:

$$L = \frac{\pi}{R-r} \left[R \sqrt{R^2 + \frac{(R-r)^2 + S_o^2}{4\pi^2}} - r \sqrt{r^2 + \frac{(R-r)^2 + S_o^2}{4\pi^2}} + \frac{(R-r)^2 + S_o^2}{4\pi^2} \ln \left(\frac{R + \sqrt{R^2 + \frac{(R-r)^2 + S_o^2}{4\pi^2}}}{r + \sqrt{r^2 + \frac{(R-r)^2 + S_o^2}{4\pi^2}}} \right) \right] \quad (27)$$

Both formula (25) and formula (26) are convenient for engineering use. They include only the values of the conical surface dimensions and the center-to-center pitch S_o . An accurate value for the length of the helical line will allow for the calculation of the axial pitch T_g to ensure the required microrelief parameters.

Conclusions

Technological schemes for forming a regular micro relief with continuous grooves on the conical surfaces of bodies of revolution have been considered. The technological features of forming a regular micro relief with triangular-shaped grooves on the conical surfaces of bodies of revolution have been established. A condition for the regularity of such grooves has been obtained depending on their geometric parameters, placement parameters, and the parameters of the conical surface. It has been established that when ensuring the regularity of the micro relief grooves, the center-to-center distance S_o is a discrete parameter proportional to the pitch of the micro relief grooves T_k .

References

- Gurey, V., Hurey, I., Hurey, T., Wojtowicz, W. (2023). Fatigue Strength of Steel Samples After Friction Treatment. In: Tonkonogyi, V., Ivanov, V., Trojanowska, J., Oborskyi, G., Pavlenko, I. (eds) *Advanced Manufacturing Processes IV. InterPartner 2022. Lecture Notes in Mechanical Engineering*. Springer, Cham. https://doi.org/10.1007/978-3-031-16651-8_26
- Hurey, I., Hurey, T., Gurey, V. (2020). Wear Resistance of Hardened Nanocrystalline Structures in the Course of Friction of Steel-Grey Cast Iron Pair in Oil-Abrasive Medium. In: Ivanov, V., et al. *Advances in Design, Simulation and Manufacturing II. DSMIE 2019. Lecture Notes in Mechanical Engineering*. Springer, Cham. https://doi.org/10.1007/978-3-030-22365-6_57
- Swirad, S. Influence of Ball Burnishing on Lubricated Fretting of the Titanium Alloy Ti6Al4V. *Lubricants* 2023, 11, 341. <https://doi.org/10.3390/lubricants11080341>
- Pawlus, P., Reizer, R., & Wiczorowski, M. (2025). Improvement of wear scar volume estimation in reciprocating motion. *Wear*, 572-573, Article 205615. <https://doi.org/10.1016/j.wear.2024.205615>.
- Pawlus, P., Koszela, W., & Reizer, R. (2022). Surface texturing of cylinder liners: A review. *Materials*, 15, 8629. <https://doi.org/10.3390/ma15238629>
- Zhan, X., Yi, P., Liu, Y., Xiao, P., Zhu, X., & Ma, J. (2020). Effects of single- and multi-shape laser-textured surfaces on tribological properties under dry friction. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 234(7), 1382–1392. <https://doi.org/10.1177/0954406219892294>
- Slavov, S., & Dimitrov, D. (2018). A study for determining the most significant parameters of the ball-burnishing process over some roughness parameters of planar surfaces carried out on CNC milling machine. In *MATEC Web of Conferences*, 178, 02005. <https://doi.org/10.1051/mateconf/201817802005>
- Dzyura, V., Maruschak, P., & Prentkovskis, O. (2021). Determining optimal parameters of regular microrelief formed on the end surfaces of rotary bodies. *Algorithms*, 14, 46. <https://doi.org/10.3390/a14020046>
- Slavov, I. Iliev, Design and FEM static analysis of an instrument for surface plastic deformation of non-planar functional surfaces of machine parts, *Fiability & Durability*, ISSN 1844 – 640X, 2016, Nov 1(2) DOI: 10.13140/RG.2.2.23594.21447.
- S. Wos, W. Koszela, P. Pawlus. Comparing tribological effects of various chevron-based surface textures under lubricated unidirectional sliding. *Tribology International* 146 (2020) 106205 : (веб-сайт). URL: <https://doi.org/10.1016/j.triboint.2020.106205>
- Jianfei Wang, Weihai Xue, Siyang Gao, Shu Li, Deli Duan. Effect of groove surface texture on the fretting wear of Ti-6Al-4V alloy. *Wear*. Vol. 486-487, 2021, 204079, <https://doi.org/10.1016/j.wear.2021.204079>.
- Stepień, P. (2009). Regular surface texture generated by special grinding process. *Journal of Manufacturing Science and Engineering*, 131(1), 011015. <https://doi.org/10.1115/1.3070511>

Дзюра В.О., Кирик С.С. Технологічні особливості формування регулярних мікрорельєфів на конічних поверхнях

В статті розглянуто особливості технологічних схем формування регулярних мікрорельєфів на конічних поверхнях. Проведено класифікацію цих схем формування за регулярністю створюваного мікрорельєфу та його геометричними параметрами. Отримано множину можливих варіантів канавок сформованого мікрорельєфу, яка передбачає 54 варіанти наборів геометричних параметрів, які поєднують форму осьової лінії канавок мікрорельєфу, закономірність зміни геометричних параметрів канавок та власне їх форму. Розглянуто особливості кожної з технологічних схем та отримано основні аналітичні залежності, що визначаються взаємозв'язок між геометричними параметрами, що визначають форму елементів канавок мікрорельєфу – кроком T_k та амплітудою A_g , параметрами розміщення канавок – міжосьовою відстанню S_o , параметрами поверхні – кутом нахилу α_c та довжиною осьової лінії регулярних мікронерівностей, що сформовані на конічній поверхні. Отримані залежності допоможуть забезпечити регулярність елементів канавок мікрорельєфу та сформувати мікрорельєф із необхідними геометричними параметрами. Встановлено, що основою подальших розрахунків елементів канавок розміщених по гвинтовій лінії конусної поверхні є визначення довжини цієї лінії. Отримано аналітичну залежність для визначення довжини гвинтової лінії конусної поверхні, як основи забезпечення регулярності канавок мікрорельєфу сформованого на гвинтовій лінії за методом геометричної подібності.

Ключові слова: регулярні мікрорельєфи, канавки, конічні поверхні, геометричні параметри