



Numerical Rheological Modeling of Thin Lubricating Oil Films Considering Polymolecular Adsorption and ZDDP

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Abstract

A numerical rheological model of a thin lubricating oil film was developed and verified to account for polymolecular adsorption, pseudo-solid adsorption layers, and the influence of zinc dialkyldithiophosphate (ZDDP) on the rheological properties of the lubricating film. The model adapts the concept of the Polanyi adsorption potential to describe the viscosity distribution across the film thickness, incorporating pseudo-solid adsorption layers adjacent to both friction surfaces, while the effective viscosity is determined by harmonic averaging with numerical integration performed using the trapezoidal rule ($n = 200$) implemented in Microsoft Excel. Shear-thinning of the bulk lubricant is described by the Carreau–Yasuda model. Model parameters were identified by fitting calculated friction coefficients to experimental data obtained for base oil I-20A and oils containing 1.25 and 2.5 wt% ZDDP. The identified adsorption layer parameters increased systematically with ZDDP concentration: surface viscosity μ_s rose from 0.14 to 1.68 Pa·s, the characteristic decay length l_d from 2.1 to 4.6 nm, and the optimum pseudo-solid layer thickness h_s from 0.8 to 1.9 nm. The average relative deviation between calculated and experimental friction coefficients was 6.9–10.2%. The calculated pressure corresponding to the minimum friction coefficient shifted from approximately 5 MPa for the base oil to about 8 MPa for ZDDP-containing oils, indicating an expansion of the hydrodynamic lubrication regime. The developed model represents a novel numerical approach that combines polymolecular adsorption, pseudo-solid adsorption layers and non-Newtonian lubricant behaviour for thin-film lubrication, suggesting that—beyond tribochemical processes—an increase in the effective viscosity of the polymolecular adsorption layer makes a significant contribution to friction reduction in the mixed lubrication regime. The proposed Excel-based methodology enables identification of model parameters and comparative prediction of lubricant tribological performance without specialized numerical software, and may be used as an engineering tool for assessing the influence of ZDDP concentration on the tribological performance of lubricating oils for plain bearings.

Keywords: thin-film lubrication, polymolecular adsorption, ZDDP, pseudo-solid adsorption layer, effective viscosity, rheological modeling, friction coefficient.

Introduction

Current developments in internal combustion engines are focused on improving energy efficiency, reducing fuel consumption, and lowering exhaust emissions. One of the principal approaches to achieving these objectives is the use of low-viscosity engine oils, with a transition from conventional SAE 40–50 grades to SAE 0W-16 and even lower viscosity grades. However, reducing the bulk viscosity of the lubricant (μ_0) inevitably decreases the thickness of the hydrodynamic lubricating film and expands the mixed lubrication regime in heavily loaded tribological contacts, particularly in connecting-rod and main journal bearings, where contact pressures may reach 10–20 MPa [1, 2].

When the lubricant film thickness decreases to the submicrometre and nanometre scales, the rheological behaviour of the lubricant may differ significantly from its bulk properties. Experimental studies based on the surface forces apparatus (SFA) and molecular dynamics simulations have demonstrated that, when the separation between solid surfaces becomes smaller than approximately six to seven molecular layers, the apparent viscosity of the confined lubricant increases by several orders of magnitude and the lubricant may exhibit pseudo-solid behaviour [3, 4]. This phenomenon, commonly referred to as confinement-induced non-Newtonian behaviour, is



generally attributed to the molecular ordering of the lubricant under adsorption forces and to the restricted molecular mobility normal to the solid surfaces.

Zinc dialkylthiophosphate (ZDDP) is one of the most widely used anti-wear additives in modern engine oils. Its effectiveness has traditionally been attributed to the formation of protective tribochemical phosphate-based films on metallic surfaces during sliding contact [5, 6]. However, several experimental studies have reported a noticeable reduction in friction at the early stages of loading or under mixed lubrication conditions, where tribofilms are not yet fully developed [7]. These observations suggest that, in addition to tribochemical reactions, other physical mechanisms associated with changes in the structure and rheological properties of the adsorption layer may contribute to friction reduction.

Despite considerable progress in thin-film lubrication research, no engineering model is currently available that simultaneously accounts for polymolecular adsorption, the formation of pseudo-solid adsorption layers, the non-Newtonian rheology of the lubricant, and the influence of ZDDP on the rheological behaviour of thin lubricating films. Addressing this gap is essential for predicting the tribological performance of modern low-viscosity engine oils operating under mixed lubrication conditions.

The aim of this study is to develop and verify a numerical rheological model of a thin lubricating oil film based on the concept of polymolecular adsorption, incorporating pseudo-solid adsorption layers and non-Newtonian lubricant behaviour, and to validate the proposed model using experimental friction data obtained for the base oil I-20A and oils containing different concentrations of ZDDP. The numerical implementation was developed in Microsoft Excel to provide an engineering-oriented methodology suitable for practical tribological analysis without specialized computational software.

Literature review

Classical elastohydrodynamic lubrication (EHL) theory, developed by Dowson and Higginson [8], assumes that the lubricant viscosity is uniformly distributed across the film thickness. This assumption provides satisfactory accuracy for conventional hydrodynamic contacts; however, its applicability decreases as the lubricant film thickness approaches the submicrometre and nanometre scales. Experimental investigations using surface forces apparatus (SFA) techniques and nanoscale rheological measurements have demonstrated a pronounced increase in the apparent viscosity of lubricants confined between closely spaced solid surfaces [3, 4, 9]. These findings indicate that the rheological behaviour of thin lubricating films differs substantially from that of the bulk lubricant and cannot be adequately described by conventional EHL models. Several approaches have been proposed to account for viscosity inhomogeneity within thin lubricating films. Tichy [10] introduced a surface-layer model based on a stepwise increase in viscosity adjacent to the solid surface, whereas Ono [11] developed a modified Reynolds equation for lubrication with a highly viscous boundary layer. These models improved the description of thin-film flow; however, they do not consider polymolecular adsorption, the symmetric formation of adsorption layers on both friction surfaces, or the influence of lubricant additives on the rheological properties of the adsorption layer. The concept of the Polanyi adsorption potential [12] provides a theoretical framework for describing polymolecular adsorption on solid surfaces. Although this concept has been widely applied to adsorption phenomena in gases and porous materials, its adaptation to thin-film lubrication offers a physically justified basis for modelling the continuous variation of lubricant viscosity within the adsorption layer. Such an approach makes it possible to represent the gradual transition from highly structured near-surface regions to the bulk lubricant instead of assuming an abrupt change in material properties. Most contemporary studies devoted to zinc dialkylthiophosphate (ZDDP) focus on the mechanisms of tribofilm formation and their influence on friction and wear [5–7, 14]. Correlations between tribofilm morphology, chemical composition, adhesion properties, and tribological performance have been extensively investigated. However, considerably less attention has been paid to the possible contribution of adsorption-induced changes in lubricant rheology to friction reduction under mixed lubrication conditions. Recent studies of confined lubricants and boundary films [13, 15, 16] indicate that molecular ordering and non-Newtonian behaviour may significantly influence friction, although these effects have rarely been incorporated into engineering lubrication models.

Therefore, the available literature indicates the absence of an engineering-oriented model that simultaneously considers polymolecular adsorption, the formation of pseudo-solid adsorption layers on both friction surfaces, the non-Newtonian rheology of the lubricant, and the influence of ZDDP on the rheological properties of thin lubricating films. Developing such a model is expected to improve the physical interpretation of friction processes in the mixed lubrication regime and provide a practical numerical tool for analysing the tribological performance of modern low-viscosity engine oils.

Problem statement

Existing thin-film lubrication models primarily describe the lubricant either as a homogeneous Newtonian continuum or by introducing simplified high-viscosity boundary layers with prescribed properties. Although such approaches improve the prediction of lubrication performance under certain operating conditions, they do not simultaneously account for polymolecular adsorption, the gradual variation of viscosity across the lubricant film,

non-Newtonian shear-thinning behaviour, and the influence of anti-wear additives on the rheological properties of the adsorption layer.

Experimental observations reported in the literature indicate that lubricant molecules confined within nanometre-scale films exhibit pronounced structural ordering, resulting in a substantial increase in apparent viscosity and the formation of highly structured adsorption layers adjacent to solid surfaces. At the same time, ZDDP additives are known not only to promote tribochemical film formation but also to modify intermolecular interactions within the lubricant. These observations suggest that changes in the rheological properties of the adsorption layer may contribute to friction reduction under mixed lubrication conditions. Based on these considerations, the lubricating film is represented as a heterogeneous medium consisting of a bulk lubricant region and adsorption layers formed on both friction surfaces. The viscosity within the adsorption layer is assumed to vary continuously according to a function derived from the concept of the Polanyi adsorption potential, while the near-surface regions are represented by pseudo-solid adsorption layers characterized by restricted molecular mobility. The rheological behaviour of the bulk lubricant is described using the Carreau–Yasuda model to account for shear-thinning effects. The objective of the proposed model is to establish a physically justified relationship between the rheological characteristics of the adsorption layer and the experimentally observed friction behaviour of lubricants with different ZDDP concentrations. The model is intended to provide an engineering-oriented numerical framework suitable for parameter identification and comparative analysis of lubricating oils operating under mixed lubrication conditions.

Theoretical Model and Numerical Calculation Methodology

To describe the friction processes in the mixed lubrication zone, the lubricating film is regarded as a non-uniform medium consisting of bulk oil and near-surface adsorption layers formed on both friction surfaces. It is assumed that, as a result of polymolecular adsorption, the viscosity of the lubricant varies continuously across the film thickness. In the immediate vicinity of the surfaces, a pseudo-solid adsorption layer is formed in which the mobility of molecules is substantially restricted. The rheological properties of the bulk oil are described by the Carreau–Yasuda model, whereas the viscosity distribution in the adsorption layer is determined by the adapted concept of the Polanyi adsorption potential [12].

It is assumed that the change in viscosity is of a continuous nature and is determined by the degree of structuring of the lubricant molecules in the adsorption layer. For the mathematical description of this process, an exponential dependence is used, which reflects the gradual decrease in the influence of the surface with increasing distance from it:

$$\mu(\zeta) = \mu_s + (\mu_0 - \mu_s) \cdot \left[1 - \exp\left(-\frac{\zeta}{l_h}\right) \right], \quad (1)$$

where ζ is the distance from the current point in the film to the nearest surface; μ_0 is the dynamic viscosity of the bulk oil; μ_s is the viscosity of the adsorption layer near the surface; l_h is the characteristic decay length of the adsorption potential.

At $\zeta = 0$, the viscosity equals μ_s . At $\zeta \gg l_h$, the viscosity approaches the bulk value μ_0 . The parameter l_h is physically interpreted as the thickness of the zone in which surface adsorption forces significantly affect the mobility of lubricant molecules. For low-molecular-weight hydrocarbon oils, l_h is 1–3 nm, whereas for oils with complex additive packages this value may reach 5–10 nm [3, 13].

For a symmetric contact with identical adsorption layers on both surfaces, the distance ζ is determined as

$$\zeta(z) = \min(z - h_s, h - h_s - z), \quad (2)$$

where z is the coordinate in the direction normal to the surface, measured from the lower boundary of the pseudo-solid layer; h is the total thickness of the lubricating film; h_s is the thickness of the pseudo-solid adsorption layer on each surface.

The effective thickness of the fluid film, in which hydrodynamic flow occurs, is

$$h_f = h - 2h_s. \quad (3)$$

The friction force is determined by shear in the fluid part:

$$F_t = \bar{\mu} \cdot A \cdot \frac{u}{h - 2h_s},$$

where A is the contact area; u is the relative sliding velocity; $\bar{\mu}$ is the effective viscosity of the film.

The optimal thickness of the pseudo-solid layer is determined from the condition of minimum friction force:

$$\frac{dF_t}{dh_s} = 0.$$

This criterion reflects the competition between two opposing trends: on the one hand, an increase in h_s reduces the effective thickness of the fluid film, which at constant $\bar{\mu}$ leads to an increase in the velocity gradient and friction force; on the other hand, pseudo-solid layers increase the effective viscosity $\bar{\mu}$ due to high-viscosity adsorption layers and enhance the hydrodynamic lifting force, which promotes an increase in h and a reduction in the velocity gradient.

The effective viscosity of the lubricating film is determined by harmonic averaging of the local viscosity over the thickness of the fluid part:

$$\bar{\mu} = \frac{h - 2h_s}{h - h_s} \cdot \ln \left(\frac{\int_{h_s}^{h-h_s} \frac{dz}{\mu(z)} \right). \quad (4)$$

Harmonic averaging is employed because individual layers of the lubricant material act as elements connected in series that resist shear deformation. This approach is widely used for multilayer media with a non-uniform distribution of rheological properties.

For a symmetric contact with the viscosity profile (1), the integral in the denominator of (4) is calculated analytically:

$$\int_{h_s}^{h-h_s} \frac{dz}{\mu(z)} = \frac{2l_h}{\mu_0} \cdot \ln \left\{ \frac{\mu_0 \cdot \exp\left(\frac{h-2h_s}{2l_h}\right) - \mu_0 + \mu_s}{\mu_s} \right\}. \quad (5)$$

In the general case, when the adsorption potentials from both surfaces overlap or the contact conditions are asymmetric, the integral is computed numerically.

Accounting for Non-Newtonian Behaviour. The rheological properties of the bulk oil are described by the Carreau–Yasuda model [17]:

$$\mu_0(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty) \cdot \left[1 + (\lambda \dot{\gamma})^a \right]^{\frac{n-1}{a}}, \quad (6)$$

where $\dot{\gamma} = u/h_s$ is the average shear rate in the fluid part of the film; λ is the relaxation time; a is the transition parameter; n is the power-law index; μ_∞ is the limiting viscosity at high shear rates. At low shear rates the model reduces to the Newtonian flow law, whereas at high shear rates it describes shear thinning of the lubricant.

The numerical implementation of the model was performed in the Microsoft Excel environment. Integration of the viscosity distribution across the thickness of the lubricating film was carried out using the trapezoidal method with $n=200$ nodes, which ensured convergence of the results. The model parameters (μ_s , l_h , h_s) were determined by minimising the root-mean-square deviation between the experimental and calculated values of the friction coefficient.

The parameters subject to identification include:

- μ_s – viscosity of the adsorption layer near the surface;
- l_h – characteristic decay length of the adsorption potential;
- h_s – thickness of the pseudo-solid adsorption layer.

The other parameters (μ_0 , λ , a , n , μ_∞) were taken in accordance with the experimental conditions and literature data for base oil I-20A.

Results of Numerical Modeling and Model Verification

The model parameters μ_s , l_h and the optimal thickness of the pseudo-solid layer $h_{s,opt}$ were identified by approximating the calculated values of the friction coefficient to the experimental data for base oil I-20A and oils containing 1.25 and 2.5 wt.% ZDDP using the least-squares method. The optimisation was carried out by the method of successive approximations in Microsoft Excel, using as the objective function the minimum of the sum of squared deviations between the calculated and experimental values of the friction coefficient. The identification results are presented in Table 1.

Table 1

Model parameters for the investigated oils			
Parameter	Base oil I-20A (0% ZDDP)	1.25 % ZDDP	2.5 % ZDDP
μ_s , Pa·s	0.14	0.98	1.68
μ_0/μ_0	5.0	35.0	60.0
l_h , nm	2.1	3.8	4.6
$h_{s,opt}$ nm	0.8	1.4	1.9

Although the identified surface viscosity is one to two orders of magnitude higher than the bulk viscosity, this parameter characterizes the rheological response of the highly structured adsorption layer rather than the bulk lubricant and therefore should not be interpreted as the viscosity of the liquid phase.

With increasing ZDDP concentration, a systematic increase in surface viscosity μ_s , characteristic decay length l_h , and optimal thickness of the pseudo-solid adsorption layer h_s is observed.

Calculation of the $\mu(\zeta)$ profile using equation (1) at a fixed bulk viscosity of $\mu_0 = 0.028$ Pa·s showed that for base oil I-20A with the fitted parameters $\mu_s = 0.14$ Pa·s and $l_h = 2.1$ nm, the viscosity reaches 95% of the bulk value at a distance of $\zeta \approx 6$ nm from the surface. For oils with ZDDP, the transition zone expands: at $\mu_s = 0.98$ Pa·s and $l_h = 3.8$ nm (1.25% ZDDP) and $\mu_s = 1.68$ Pa·s and $l_h = 4.6$ nm (2.5% ZDDP), the viscosity approaches μ_0 only at a distance of 15–20 nm. This indicates the formation of a thicker and denser polymolecular adsorption layer upon the introduction of ZDDP, which is consistent with data on the formation of phosphate adsorption complexes on iron surfaces [5, 13].

Figure 1 presents a comparison of the experimental and calculated dependences of the friction coefficient on contact pressure.

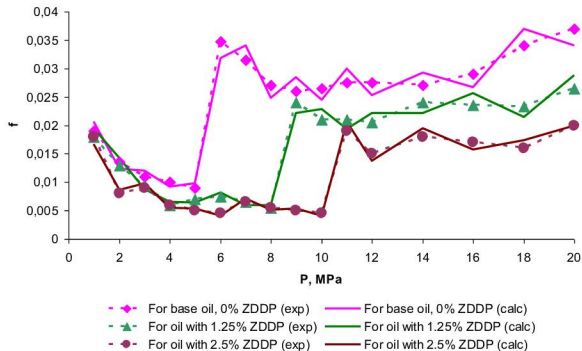


Fig. 1. Dependence of the friction coefficient on contact pressure for base oil and oils with various ZDDP concentrations

For all investigated oils, the model correctly reproduces the shape of the curves, the position of the friction coefficient minimum, and the character of the transition between lubrication regimes.

The average relative deviation between calculated and experimental values was 10.2% for the base oil, 7.8% for the oil with 1.25% ZDDP, and 6.9% for the oil with 2.5% ZDDP.

Based on the experimental and calculated $f(P)$ curves, the characteristic points of the Stribeck diagram have been determined (Table 2). The data are presented as calculated examples typical for similar tribosystems.

The pressure corresponding to the minimum friction coefficient shifts towards higher values for oils containing ZDDP. The hydrodynamic lubrication range, defined as the interval from the minimum to the onset of the sharp increase in friction, expands upon introduction of the additive. The relative expansion of the hydrodynamic lubrication range is approximately 60% for 1.25% ZDDP and 100% for 2.5% ZDDP compared with the base oil.

For the oil with 2.5% ZDDP, the friction minimum is flatter (a plateau in the range of 6–10 MPa), indicating an extended zone of stable hydrodynamic lubrication due to the formation of a thick adsorption–tribochemical layer with enhanced load-carrying capacity.

Table 2

Characteristic points of the Stribeck diagram

Parameter	Base oil I-20A (0% ZDDP)	1.25 % ZDDP	2.5 % ZDDP
P_1 (minimum f), MPa	4–6	6–10	6–10
$f_{\min}$	0.008–0.010	0.005–0.007	0.004–0.006
P_2 (start of f increase), MPa	5–7	8–10	9–12
P_3 (boundary regime plateau), MPa	9–12	12–16	14–18

The obtained parameters demonstrate a monotonic increase with growing ZDDP concentration. The most substantial growth is observed for surface viscosity μ_s : the ratio μ_s/μ_0 changes from 5.0 for the base oil to 35.0 and 60.0 for oils with ZDDP. This indicates enhanced structuring of the lubricant material in near-surface layers upon introduction of the antiwear additive.

To generalise the results of numerical modelling, dependences of the effective viscosity of the lubricating film on its thickness have been constructed for the base oil and oils with various ZDDP concentrations (Fig. 2). The calculated dependences show that at a film thickness $h < 0.3 \mu\text{m}$, the effective viscosity of oils with ZDDP exceeds the corresponding value for the base oil by a factor of 2–4. As the film thickness increases, this effect gradually decreases: at $h > 1 \mu\text{m}$ the difference is only 5–10%, and at $h > 2 \mu\text{m}$ all curves practically coincide with the bulk viscosity of the oil $\mu_0 = 0.028 \text{ Pa}\cdot\text{s}$. The obtained results indicate that adsorption effects are decisive at a lubricating film thickness of less than approximately $0.3 \mu\text{m}$ for the base oil and $0.6\text{--}0.8 \mu\text{m}$ for oils with ZDDP.

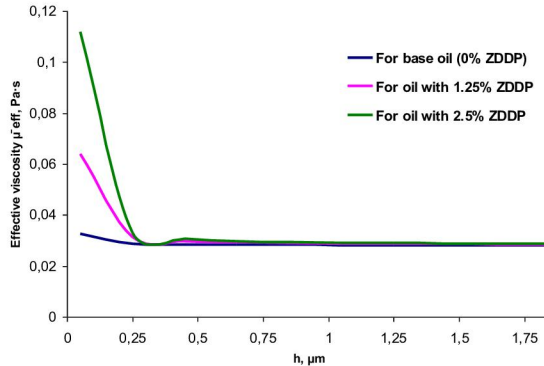


Fig. 2. Calculated dependence of the effective viscosity on the lubricating film thickness for the base oil and oils containing different ZDDP concentrations

A parametric analysis has been conducted of the influence of the characteristic length of the adsorption layer (l_a) and the surface viscosity (μ_s) on the effective viscosity of the lubricating film and the friction coefficient. At a lubricating film thickness of $h = 0.5 \mu\text{m}$, an increase in l_a from 1.0 to 6.0 nm at a fixed value of $\mu_s = 0.98 \text{ Pa}\cdot\text{s}$ leads to an increase in the effective viscosity from 0.033 to $0.061 \text{ Pa}\cdot\text{s}$. At $h = 5 \mu\text{m}$ the influence of this parameter decreases substantially: the effective viscosity exceeds the bulk viscosity μ_0 by only 1.8% and 28.6%, respectively. Thus, the sensitivity of the model to the parameter l_a is greatest in the mixed lubrication region ($0.1 < h < 1.0 \mu\text{m}$), where adsorption phenomena determine the rheological properties of the lubricating film.

A sensitivity analysis performed using normalised partial derivatives at the operating point ($P = 8 \text{ MPa}$, ZDDP concentration 1.25 wt.%) showed that the surface viscosity μ_s (normalised sensitivity coefficient -0.65) and the characteristic length of the adsorption layer l_a (-0.48) have the greatest influence on the friction coefficient, whereas the influence of the bulk viscosity μ_0 is less pronounced (-0.28). The obtained results indicate that the tribotechnical characteristics of the oil under thin-film mixed lubrication conditions are determined to a considerable extent by the structural-rheological properties of the adsorption layer, and not solely by the viscosity of the base oil.

The average relative deviation between calculated and experimental values of the friction coefficient was 6.9–10.2%. The largest deviations were observed in the region of transition from mixed to hydrodynamic lubrication regime, which may be associated with the simplified assumptions of the model regarding the properties of the adsorption layer.

The proposed model is based on a one-dimensional description of viscosity distribution and does not account for local temperature non-uniformities, elastic deformation of surfaces, and the kinetics of tribochemical growth of ZDDP films. The obtained results should be regarded as a description of the rheological contribution of polymolecular adsorption to the change in the friction coefficient. Further development of the model may be associated with accounting for elasto-hydrodynamic effects, temperature dependence of adsorption layer parameters, and tribochemical kinetics of protective film formation.

Conclusions

1. A numerical rheological model of a thin lubricating oil film has been developed based on the concept of polymolecular adsorption. The model combines a continuous viscosity distribution within the adsorption layer, pseudo-solid adsorption layers adjacent to both friction surfaces, and the non-Newtonian rheology of the bulk lubricant described by the Carreau–Yasuda model.

2. The proposed methodology enables identification of the adsorption-layer parameters from experimental friction data. The identified parameters demonstrate a systematic increase with increasing ZDDP concentration, indicating enhanced molecular structuring of the lubricant in the near-surface region.

3. Comparison between calculated and experimental friction coefficients confirmed that the proposed model adequately reproduces the characteristic shape of the friction–pressure relationship over the investigated range of contact pressures. The obtained agreement indicates that the model captures the principal rheological features governing friction under mixed lubrication conditions.

4. The obtained results suggest that, in addition to the well-established tribochemical action of ZDDP, adsorption-induced changes in the rheological properties of the lubricating film may contribute to friction reduction. The proposed model therefore provides a complementary physical interpretation of the influence of anti-wear additives on tribological behaviour.

5. The developed numerical approach is implemented in Microsoft Excel and does not require specialized computational software. Owing to its engineering-oriented formulation, the model may be applied for comparative evaluation of lubricant formulations, identification of adsorption-layer parameters, and preliminary prediction of tribological performance in plain bearings operating under mixed lubrication conditions.

Future work will focus on incorporating elasto-hydrodynamic effects, temperature-dependent rheological properties, and tribochemical film growth kinetics to extend the applicability of the proposed model to a wider range of operating conditions.

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Драч І., Диха М., Товстюк Д. Чисельне реологічне моделювання тонкого шару моторної оливи з урахуванням полімолекулярної адсорбції та впливу ZDDP

Розроблено та верифіковано чисельну реологічну модель тонкого шару моторної оливи з урахуванням полімолекулярної адсорбції, псевдотвердих адсорбційних шарів і впливу протизносної присадки ZDDP на реологічні властивості змазувального шару. Для опису розподілу в'язкості в товщині змазувального шару адаптовано реологічну модель, побудовану на основі концепції адсорбційного потенціалу Полані. До моделі введено псевдотверді адсорбційні шари на обох поверхнях тертя, а ефективну в'язкість визначено методом гармонійного усереднення з чисельним інтегруванням методом трапецій ($n = 200$), реалізованим у Microsoft Excel. Ньютонівське зрідження мастила враховано за моделлю Карро–Ясуді. Параметри моделі визначали шляхом узгодження розрахункових і експериментальних значень коефіцієнта тертя для базової оливи I-20A та оливи із вмістом ZDDP 1,25 і 2,5 мас. %. Встановлено зростання параметрів адсорбційного шару зі збільшенням концентрації ZDDP: в'язкість приповерхневого шару μ_s збільшилася від 0,14 до 1,68 Па·с, характерна довжина спаду h_s – від 2,1 до 4,6 нм, а оптимальна товщина псевдотвердого шару h_p – від 0,8 до 1,9 нм. Середнє відносне відхилення між розрахунковими та експериментальними значеннями коефіцієнта тертя становило 6,9–10,2 %, що підтверджує задовільну узгодженість моделі з експериментальними даними. Розрахунковий тиск, що відповідає мінімуму коефіцієнта тертя, змінюється приблизно з 5 МПа для базової оливи до близько 8 МПа для оливи із ZDDP, що свідчить про розширення області гідродинамічного мащення. Наукова новизна. Розроблена чисельна реологічна модель тонкого змазувального шару є новим числовим підходом, що поєднує полімолекулярну адсорбцію, концепцію псевдотвердих адсорбційних шарів і ньютонівську реологію мастила. Отримані результати свідчать, що поряд із трибохімічними процесами істотний внесок у зниження тертя в зоні змішаного мащення може забезпечувати підвищення ефективної в'язкості полімолекулярного адсорбційного шару. Запропонована методика, реалізована в середовищі Microsoft Excel, дає змогу ідентифікувати параметри моделі та прогнозувати триботехнічні характеристики мастильних матеріалів без використання спеціалізованого програмного забезпечення. Розроблений підхід може бути використаний для інженерного оцінювання впливу концентрації ZDDP на триботехнічні властивості моторних оливи для підвищених ковзання.

Ключові слова: тонкошарове мащення, полімолекулярна адсорбція, ZDDP, псевдотвердий адсорбційний шар, ефективна в'язкість, реологічне моделювання, коефіцієнт тертя.