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TOOL REINFORCED WORKFACES WEAR FORECAST ON MATLAB MODULE "TRWWF" FOR PLOTTING WEAR STOCHASTIC PROCESS BUNDLE REALIZATIONS UNDER EVALUATIONS OF LINEAR-PARABOLIC WEAR FRACTURE AND WEAR STOCHASTIC COMPONENT MAGNITUDE

Essentiality of forecasting the wear of tool reinforced workfaces

Going into particulars, tools with reinforced workfaces are often used in agricultural machinery for plowing grounds in and out. If a tool workface is reinforced then its wear $w(t)$ through time $t \in [0; T]$ with the total exploitation expiration (TEE) T increases slower than for the nonreinforced workfaces. Nevertheless, there is a point $t_{\text{fr}} \in (0; T)$ of the fracture (figure 1), after which the wear increment accelerates through time $t \in [t_{\text{fr}}; T]$ sharply, what is conditioned with destruction of the reinforced layers and reducing down to the non-reinforced layer. Forecasting the wear of tool reinforced workfaces (TRW) helps in groping for the moment $t_{\text{fr}} \in (0; T)$ and, further, in controlling the being faded tool workface.

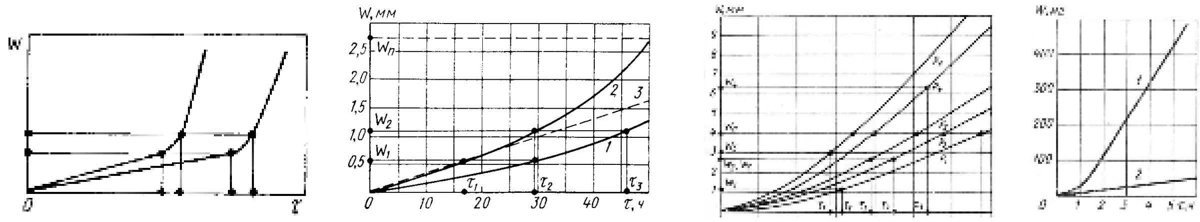

Figure 1 – Qualitative dependences $w(t)$ for TRW [1, p. 20, 29, 37, 140]

A survey over wear forecasting framework origins to TRW

It is obvious [2] the expected wear of the reinforced workface over $t \in [0; t_{\text{fr}})$ is linear, whereupon it may be described with the upward-right-branch parabolic function [1], defined on the segment $[t_{\text{fr}}; T]$ respectively. To that the simplest equation of the wear kinetics may be stated as [3]

$$dw(t) = a_w(t)dt + \lambda_w d\psi(t) \tag{1}$$

for the wear intensity $a_w(t)$ and a constant magnitude λ_w of the stochastic component of the wear, being generated up with the standard Wiener process $\{\psi(t)\}_{t\in[0;\,T]}$. Then with knowing the initial wear condition (IWC)

$$w(0) = w_0 \tag{2}$$

there is the equivalent statement of the equation (1) in the form of the stochastic process (SP)

$$w(t) = w_0 + \int_0^t a_w(\tau)d\tau + \lambda_w \int_0^t d\psi(\tau), \;\; t \in [0; T]. \tag{3}$$

An SP-solution form (3) for TRW requires only parameters of the constant and linear parts in $a_w(t)$, where the constant magnitude λ_w is laid according to the rate of the ground roughness.

A task for forecasting the wear of TRW as SP (3)

The being sprung up investigation task lies in forecasting post-numerically the wear of TRW as SP (3) with a program means, accepting inputs $\{a_w(t), \lambda_w\}$ under known TEE T and IWC (2). For this there should be coded a MATLAB environment function-solver (module-solver), returning SP (3) samples and visualizing them.

Forecasting the wear of TRW as SP (3) on MATLAB module-solver “trwwf”

It is useful firstly to restate SP $\{\psi(t)\}_{t\in[0;\,T]}$ over the normally distributed variate Ξ with zero expectation and unit variance for its values $\{\xi(t)\}_{t\in[0;\,T]}$ as [3]

$$\psi(t)=\xi(t)\sqrt{t}\,, \tag{4}$$

obtaining the standard Wiener process differential

$$d\psi(t)=\xi\sqrt{dt}\,. \tag{5}$$

Then with (5), taken L MATLAB-generated samples

$$\left\{\xi_j=\xi(t_j)\right\}_{j=1}^{L}\quad \forall\ t_{i+1}=t_i+\frac{T}{L}\ \text{ for }\ i=\overline{1,\,L-1}\ \text{ at }\ t_1=\frac{T}{L}\ \text{ and }\ t_L=T \tag{6}$$

of SP Ξ, the SP-solution form (3) for TRW is sampled as

$$w(t_j)=w_0+\frac{T}{L}\sum_{i=1}^{j}a_w(t_i)+\sqrt{\frac{T}{L}}\lambda_w\sum_{i=1}^{j}\xi_i\ \text{ for }\ j=\overline{1,\,L}\,. \tag{7}$$

Will visualize samples (7) with IWC (2) on the background of the wear expectation (WE)

$$\mathbf{M}\left[w(t)\right]=w_0+\int_0^t a_w(\tau)\,d\tau\ \text{ by }\ t\in[0;\,T], \tag{8}$$

which may be written over its linear and parabolic branches, being jointed on the point $t=t_{\text{fr}}$, that is

$$\mathbf{M}\left[w(t)\right]=w_0+\mu t\cdot\frac{1-\operatorname{sign}(t-t_{\text{fr}})}{2}+\left(\alpha t^2+\beta t+\gamma\right)\cdot\frac{1+\operatorname{sign}(t-t_{\text{fr}})}{2} \tag{9}$$

for some coefficients μ, α, β, γ. If the time-point t_{fr} WE evaluation is $\mathbf{M}\left[w(t_{\text{fr}})\right]=w_{\text{fr}}$ then

$$\mathbf{M}\left[w(t_{\text{fr}})\right]=w_{\text{fr}}=w_0+\mu t_{\text{fr}}=w_0+\alpha(t_{\text{fr}})^2+\beta t_{\text{fr}}+\gamma,\ \ \mu=\frac{\alpha(t_{\text{fr}})^2+\beta t_{\text{fr}}+\gamma}{t_{\text{fr}}}=\frac{w_{\text{fr}}-w_0}{t_{\text{fr}}} \tag{10}$$

from (9), and coefficients $\{\alpha,\,\beta,\,\gamma\}$ are found from the linear algebraic equations system (LAES)

$$w_0+\alpha(t_{\text{fr}})^2+\beta t_{\text{fr}}+\gamma=w_{\text{fr}}\,, \tag{11}$$

$$w_0+\alpha(t_{\text{par}})^2+\beta t_{\text{par}}+\gamma=\mathbf{M}\left[w(t_{\text{par}})\right]=w_{\text{par}}\,, \tag{12}$$

$$w_0+\alpha T^2+\beta T+\gamma=\mathbf{M}\left[w(T)\right]=1 \tag{13}$$

for the given post-fractured parabolic WE evaluation w_{par} (12) on the time-point $t=t_{\text{par}}$, and the total wear-out expectation (13) on TEE. The solution $\{\alpha,\,\beta,\,\gamma\}$ of LAES (11) — (13) is acceptable if

$$-\frac{\beta}{2\alpha}<t_{\text{fr}} \tag{14}$$

and

$$\frac{d}{dt}(w_0+\mu t)<\frac{d}{dt}\left(\alpha t^2+\beta t+\gamma\right)\quad \forall\ t\in[t_{\text{fr}};\,T], \tag{15}$$

that is

$$\mu<2\alpha t+\beta\quad \forall\ t\in[t_{\text{fr}};\,T] \tag{16}$$

from (15). The conditions (14) and (16) are included into MATLAB module-solver “trwwf”, acquiring its eight input arguments T, w_0, t_{fr}, w_{fr}, t_{par}, w_{par}, λ_w, L, generating samples (6), solving LAES (11) — (13) into the solution $\{\alpha,\,\beta,\,\gamma\}$ and returning samples (7) concurrently with samples $\left\{\mathbf{M}\left[w(t_j)\right]\right\}_{j=1}^{L}$ of WE (9), also plotting them immediately. The figures 2 — 5 with plotted bundles of the wear SP of TRW elucidate patterns of how to run the module “trwwf” properly, at that adjusting parameters of the time-wear dependence.

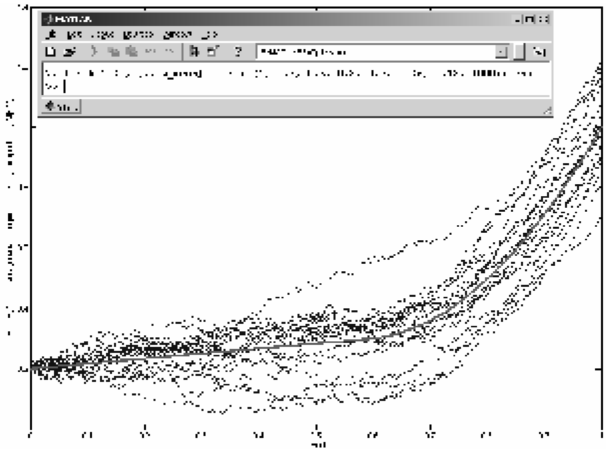

Figure 2 – A bundle of the wear SP (3) 20 realizations of TRW as its samples (7) with samples $\{\mathrm{M}\ w(t_l)\}_{l=1}^{L}$ of WE (9) by $T=1$, $w_0=0.2$, $t_{\mathrm{fr}}=0.6$, $w_{\mathrm{fr}}=0.3$, $t_{\mathrm{par}}=0.7$, $w_{\mathrm{par}}=0.36$, $\lambda_w=0.12$, $L=1000$

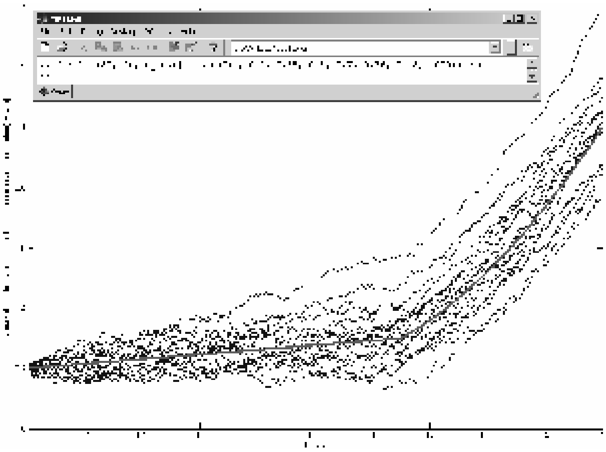

Figure 3 – A bundle of the wear SP (3) 20 realizations of TRW as its samples (7) with samples $\{\mathrm{M}\ w(t_l)\}_{l=1}^{L}$ of WE (9) by $T=1$, $w_0=0.2$, $t_{\mathrm{fr}}=0.65$, $w_{\mathrm{fr}}=0.3$, $t_{\mathrm{par}}=0.7$, $w_{\mathrm{par}}=0.36$, $\lambda_w=0.12$, $L=1000$

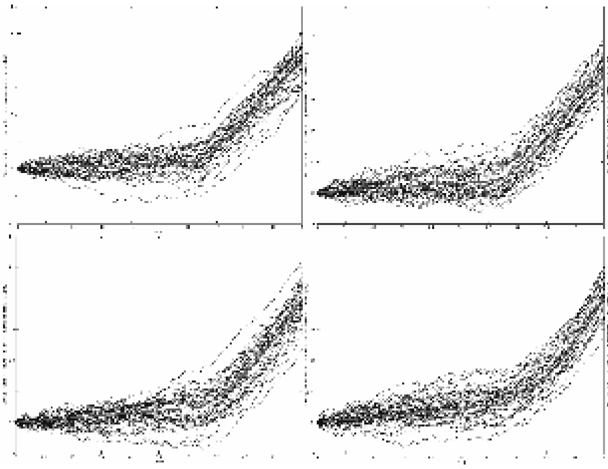

Figure 4 – Adjusting the time-point t_{fr} WE evaluation of the wear SP (3) of TRW, where $w_{\mathrm{fr}} \in \{0.26, 0.28, 0.30, 0.32\}$ under the previous values of the rest of parameters

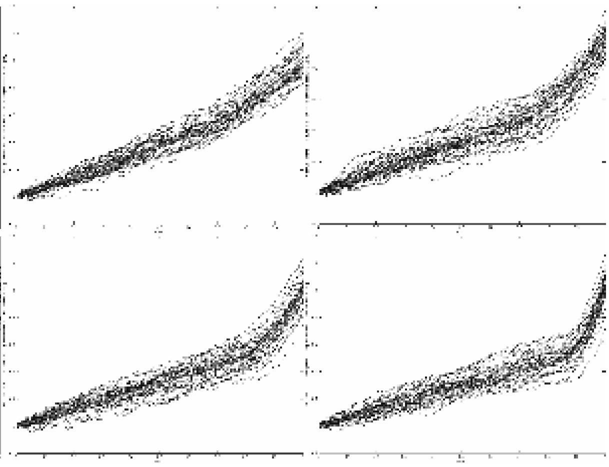

Figure 5 – Adjusting the time-points $t_{\mathrm{fr}} \in \{0.66, 0.75, 0.8, 0.87\}$ and $t_{\mathrm{par}} \in \{0.7, 0.79, 0.84, 0.9\}$ by $T=1$, $w_0=0$, $w_{\mathrm{fr}}=0.5$, $w_{\mathrm{par}}=0.55$, $\lambda_w=0.1$, $L=1000$

Completion and outlining the further investigation

The program MATLAB means, represented with its employ above, may be useful also in determining t_{fr}, w_{fr}, t_{par}, w_{par} by adjusting WE (8) as (9) under two or more measurements. Then, having adjusted WE (8) as (9), an investigator may observe bundles of wear SP realizations and mark its stretching width and the wear process upper envelope in the bundle, which indicate roughly the wear worst case that could flow. By that certainly the wear SP values, exceeding the bounds of $[0; 1]$, should be ignored. And the further investigation might be connected with forecasting wear as the function of time and location.

References

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