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A THEOREM ON CONTINUUM OF THE PROJECTOR OPTIMAL BEHAVIORS IN ANTAGONISTIC MODEL OF BUILDING RESOURCES DISTRIBUTION UNDER SEGMENT UNCERTAINTIES WITH INCORRECTLY PRE-EVALUATED ONE LEFT AND ONE RIGHT ENDPOINTS

Introduction and the problem general description

Because of different antagonistic factors in building, there is a problem of indeterminancies elimination in projecting mount design constructions [1]. To resolve them at least as possible, there is a known convex antagonistic game (AG) with the kernel

$$T(\mathbf{X}, \mathbf{Y}) = T(x_1, x_2, x_3; y_1, y_2, y_3) = \max\left(\left\{r_i(x_i, y_i)\right\}_{i=1}^{4}\right) =$$

$$= \max\left\{x_1 y_1^{-2}, x_2 y_2^{-2}, x_3 y_3^{-2}, \frac{1 - x_1 - x_2 - x_3}{(1 - y_1 - y_2 - y_3)^2}\right\} \quad (1)$$

on the product

$$\boldsymbol{X} \times \boldsymbol{Y} = \prod_{p=1}^{2} [a_1; b_1] \times [a_2; b_2] \times [a_3; b_3] =$$

$$= \prod_{p=1}^{2} \left(\prod_{d=1}^{3} [a_d; b_d] \right) \subset \prod_{i=1}^{6} (0; 1) \subset \prod_{i=1}^{6} [0; 1] \subset \mathbb{R}^6 \quad (2)$$

of the parallelepiped

$$\boldsymbol{X} = [a_1; b_1] \times [a_2; b_2] \times [a_3; b_3] = \prod_{d=1}^{3} [a_d; b_d] \subset \prod_{d=1}^{3} (0; 1) \subset \prod_{d=1}^{3} [0; 1] \subset \mathbb{R}^3 \quad (3)$$

of pure strategies

$$\mathbf{X} = [x_1 \; x_2 \; x_3] \in [a_1; b_1] \times [a_2; b_2] \times [a_3; b_3] = \boldsymbol{X} \quad (4)$$

of the first player and of the parallelepiped

$$\boldsymbol{Y} = [a_1; b_1] \times [a_2; b_2] \times [a_3; b_3] = \prod_{d=1}^{3} [a_d; b_d] \subset \prod_{d=1}^{3} (0; 1) \subset \prod_{d=1}^{3} [0; 1] \subset \mathbb{R}^3 \quad (5)$$

of pure strategies

$$\mathbf{Y} = [y_1 \; y_2 \; y_3] \in [a_1; b_1] \times [a_2; b_2] \times [a_3; b_3] = \boldsymbol{Y} \quad (6)$$

of the second player, where variables x_4 and y_4 due to

$$\sum_{i=1}^{4} x_i = 1, \; \sum_{i=1}^{4} y_i = 1 \quad (7)$$

are excluded. However, for some specific cases of the endpoints $\{a_d\}_{d=1}^{3}$ and $\{b_d\}_{d=1}^{3}$ pre-evaluation, there may be complicated the known minimax procedure for finding the optimal behavior of the second player

$$\mathbf{Y}_* = [y_1^* \; y_2^* \; y_3^*] \subset [a_1; b_1] \times [a_2; b_2] \times [a_3; b_3] = \boldsymbol{Y} \quad (8)$$

as the projector.

The origins survey and the unsolved question underlining

The mentioned minimax procedure [2, 3] for finding the optimal behavior of the second player as its optimal pure strategy (8) issues from the theorem on the second player pure strategies in the convex antagonistic game [4, 5]. Mainly the components of the optimal pure strategy (8) satisfy the four-parted equality

$$v_* = b_1 \left(y_1^* \right)^{-2} = b_2 \left(y_2^* \right)^{-2} = b_3 \left(y_3^* \right)^{-2} = \frac{1 - a_1 - a_2 - a_3}{\left(1 - y_1^* - y_2^* - y_3^* \right)^2}, \tag{9}$$

but sometimes, because of the incorrect pre-evaluation of the endpoints $\{a_d\}_{d=1}^{3}$ and $\{b_d\}_{d=1}^{3}$, the equality (9) cannot be reached within the parallelepiped (5). One of the most interesting cases is that when there are only two endpoints from those six, having been pre-evaluated incorrectly, but the first is right, and the second is left.

Paper aim

Being guided by the supposition of that there are only two endpoints from those six, having been pre-evaluated incorrectly, but the first is right, and the second is left, will solve the AG with kernel (1) on the product (2) for the projector. For making it there must be conditioned those incorrectly pre-evaluated endpoints, so that they are a_p and b_q by $p \in \{\overline{1, 3}\}$ and $q \in \{\overline{1, 3}\}$ for $p \neq q$.

A theorem on the continuum of the projector optimal strategies in AG with kernel (1) on the product (2)

If there were the four-parted equality (9) within parallelepiped (5), then the components $\{y_d^*\}_{d=1}^{3}$ of the optimal pure strategy (8) would be the following [2]:

$$y_j^* = \frac{\sqrt{b_j}}{\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d}} \quad \forall\ j = \overline{1, 3}. \tag{10}$$

But here is that

$$\frac{\sqrt{b_p}}{\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d}} < a_p \quad \text{by} \quad p \in \{\overline{1, 3}\} \tag{11}$$

and

$$\frac{\sqrt{b_q}}{\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d}} > b_q \quad \text{by} \quad q \in \{\overline{1, 3}\}. \tag{12}$$

So, the component y_p^* is greater than expected before checking (9), and the component y_q^* is lesser than expected before checking (9). Then there stands the inequality

$$\frac{1}{b_q} > \frac{b_k}{\left(y_k^* \right)^2} > \frac{b_p}{a_p^2} \quad \text{at} \quad k \in \{\overline{1, 3}\} \setminus \{p, q\} \tag{13}$$

for the component $y_k^* \in [a_k; b_k]$ by (10). Due to this here is a theorem on the components $\{y_d^*\}_{d=1}^{3}$.

Theorem. In AG with kernel (1) on the product (2) by the conditions (11) and (12) at the inequality

$$\frac{1}{b_q} \geqslant \frac{1 - \sum_{d=1}^{3} a_d}{\left(1 - b_q - a_p - y_k^* \right)^2} > \frac{b_k}{\left(y_k^* \right)^2} > \frac{b_p}{a_p^2} \quad \text{at} \quad k \in \{\overline{1, 3}\} \setminus \{p, q\} \tag{14}$$

for the component $y_k^* \in (a_k; b_k]$ by (10) the projector has the continuum of its pure optimal behaviors as (8) with the component

$$y_q^* = b_q. \tag{15}$$

In more detail, if

$$1-b_q-\sqrt{b_q\left(1-\sum_{d=1}^{3}a_d\right)}\geqslant b_p+b_k \tag{16}$$

then

$$y_p^*\in\left[a_p;\,b_p\right], \tag{17}$$

$$y_k^*\in\left[\frac{1}{2}\left(\sqrt{b_qb_k}+a_k+\left(\sqrt{b_qb_k}-a_k\right)\operatorname{sign}\left(\sqrt{b_qb_k}-a_k\right)\right);\,b_k\right]. \tag{18}$$

Otherwise, if

$$1-b_q-\sqrt{b_q\left(1-\sum_{d=1}^{3}a_d\right)}<b_p+b_k \tag{19}$$

then

$$y_p^*\in\left[a_p;\,y_p^{\langle\max\rangle}\right], \tag{20}$$

$$y_k^*\in\left[\frac{1}{2}\left(\sqrt{b_qb_k}+a_k+\left(\sqrt{b_qb_k}-a_k\right)\operatorname{sign}\left(\sqrt{b_qb_k}-a_k\right)\right);\,y_k^{\langle\max\rangle}\right] \tag{21}$$

by

$$y_p^{\langle\max\rangle}+y_k^{\langle\max\rangle}=1-b_q-\sqrt{b_q\left(1-\sum_{d=1}^{3}a_d\right)}, \tag{22}$$

$$y_p^{\langle\max\rangle}\in\left[a_p;\,b_p\right], \tag{23}$$

$$y_k^{\langle\max\rangle}\in\left[\frac{1}{2}\left(\sqrt{b_qb_k}+a_k+\left(\sqrt{b_qb_k}-a_k\right)\operatorname{sign}\left(\sqrt{b_qb_k}-a_k\right)\right);\,b_k\right]. \tag{24}$$

Proof. The pre-conditioned inequality (14) means that the optimal game value $v_*=\frac{1}{b_q}$, whence the component (15) is cleared. For saving this optimal game value the projector should hold at such components y_p^* and y_k^* that there would be inequalities

$$\frac{1}{b_q}\geqslant\frac{1-\sum_{d=1}^{3}a_d}{\left(1-b_q-y_p^*-y_k^*\right)^2}, \tag{25}$$

$$\frac{1}{b_q}\geqslant\frac{b_k}{\left(y_k^*\right)^2}, \tag{26}$$

$$\frac{1}{b_q}\geqslant\frac{b_p}{\left(y_p^*\right)^2}. \tag{27}$$

Now will solve the inequality (25) with respect to the sum $y_p^*+y_k^*$. Here is

$$\left(1-b_q\right)^2-2\left(1-b_q\right)\left(y_p^*+y_k^*\right)+\left(y_p^*+y_k^*\right)^2\geqslant b_q\left(1-\sum_{d=1}^{3}a_d\right), \tag{28}$$

$$\left(y_p^*+y_k^*\right)^2-2\left(1-b_q\right)\left(y_p^*+y_k^*\right)-\left(1-b_q\right)^2-b_q\left(1-\sum_{d=1}^{3}a_d\right)\geqslant 0, \tag{29}$$

where the corresponding quadratic equation

$$\left(y_p^* + y_k^*\right)^2 - 2\left(1-b_q\right)\left(y_p^* + y_k^*\right) + \left(1-b_q\right)^2 - b_q\left(1-a_1-a_2-a_3\right) = 0 \quad (30)$$

discriminant is

$$D = 4\left(1-b_q\right)^2 - 4\left[\left(1-b_q\right)^2 - b_q\left(1-a_1-a_2-a_3\right)\right] = 4b_q\left(1-a_1-a_2-a_3\right). \quad (31)$$

The roots of the equation (30) with respect to the unknown sum $y_p^* + y_k^*$ are

$$\frac{2\left(1-b_q\right)-\sqrt{D}}{2} = \frac{2\left(1-b_q\right)-2\sqrt{b_q\left(1-a_1-a_2-a_3\right)}}{2} = 1-b_q-\sqrt{b_q\left(1-a_1-a_2-a_3\right)} \quad (32)$$

or

$$\frac{2\left(1-b_q\right)+\sqrt{D}}{2} = \frac{2\left(1-b_q\right)+2\sqrt{b_q\left(1-a_1-a_2-a_3\right)}}{2} = 1-b_q+\sqrt{b_q\left(1-a_1-a_2-a_3\right)}. \quad (33)$$

Then, inequality (25) or (29) is true by (16) or by

$$y_p^* + y_k^* \geqslant 1-b_q+\sqrt{b_q\left(1-a_1-a_2-a_3\right)}. \quad (34)$$

Though, with taking the root (33) there is

$$1-b_q-y_p^*-y_k^* = 1-b_q-\left(1-b_q+\sqrt{b_q\left(1-a_1-a_2-a_3\right)}\right) = -\sqrt{b_q\left(1-a_1-a_2-a_3\right)} < 0, \quad (35)$$

that is the root (33) cannot be the sum $y_p^* + y_k^*$ and it must be ignored, and the inequality (34) will never be true. This means that under the condition (16) the components y_p^* and y_k^* may be taken in such a way, that the inequality

$$y_k^* \geqslant \sqrt{b_q b_k} \quad \text{at} \quad k \in \left\{\overline{1, 3}\right\} \setminus \{p, q\} \quad (36)$$

would be satisfied, what gives (18) from (26). And here (17) issues from (16) and the inequality (27), being true for $y_p^* \geqslant a_p$. On the other hand, under the condition (19) the maximal sum of the components y_p^* and y_k^* must be equal to the left member in (19), satisfying the inequality (36). And this gives (20) and (21) under the sum (22) for (23) and (24). The theorem has been proved.

Conclusion and the further investigation outlook

The proved theorem under local condition (14) allows finding instantly the continuum of projector optimal behaviors, when in uncertainty segments $\left\{\left[a_d; b_d\right]\right\}_{d=1}^{3}$ one of the left endpoints and one of the right endpoints were pre-evaluated incorrectly. And if the projector does not want to re-evaluate those endpoints of uncertainty segments, then the outlook of the investigation lies in extracting the most practicable or rational optimal behaviors from their continuum with components (15), (17), (18) or (15), (20), (21) by (22) — (24) to form the unique variant of the projector decision.

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