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Heat and mass transfer models at boundary lubrication to determine the transition temperatures

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Abstract

At present, kinetic and thermodynamic methods for assessing the lubricating effect of oils are being increasingly developed. At the limit friction, the reduction of friction and wear of surfaces is due to the ability of the lubricant to form layers of adsorption or chemical origin on the surface. Analytical models of transition temperatures and wear in the limit lubrication mode must be used to mathematically describe the processes in the subsystems and the transition between them. The Fourier equation of thermal conductivity is accepted as the basic calculated dependence. It is assumed that the process of heat propagation under the conditions of formation of lubricating films is not Markovian, i.e. the magnitude of the heat flux is determined by the entire "history" of heat transfer in a certain elementary volume. The equation of motion of a lubricating film over the surface of a body that is being lubricated is obtained from the equation of motion for a Newtonian continuous medium. As a result, nonlinear heat and mass transfer models are obtained to determine the transition temperatures in the formation of boundary lubricating films in the concept of structural-thermodynamic approaches to describe the processes of boundary lubrication of surfaces.

Key words: maximum lubrication, transition temperatures, mathematical model.

Introduction

At ultimate friction, the reduction of friction and wear of surfaces is due to the ability of the lubricant to form layers of adsorption or chemical origin on the surface. At the first stage (fig.1) of interaction of the surface and the lubricant, the adsorbed layer is formed by molecules of the surfactant component. Adsorbed films under certain conditions have the ability to self-organize, ie there is a process of dynamic equilibrium between the formation and destruction of adsorbed layers under the action of external factors. When the value of the first transition temperature is reached, the dynamic equilibrium is disturbed and the adsorption layers are destroyed. Most modern lubricants contain chemically active components, which at the temperature of chemical modification form chemically modified films on the metal surface, which leads to reduced friction and wear. Further increase of external influence leads to complete destruction of the boundary lubricating layer at the second critical temperature.

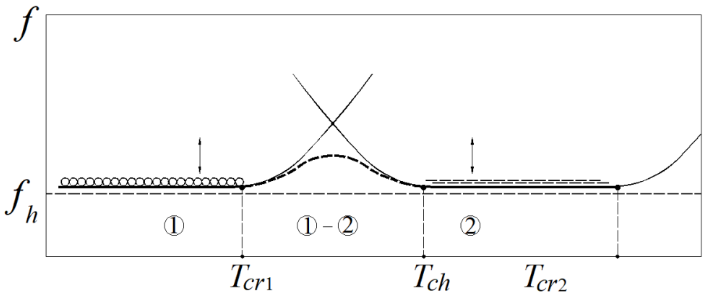

Fig.1. The kinetics of the formation of transition temperatures.

Analytical models of transition temperatures and wear in the limit lubrication mode must be used to mathematically describe the processes in the subsystems and the transition between them.

Literatuter review

Much attention is currently being paid to the study of models of heat and mass transfer during boundary lubrication. The heat transfer phenomenon is beneficial and applicable in engineering, industries, and technological processes. The production of energy with the help of some cheap resources plays a pivotal and renewable role in the industrial development of the countries. In study [1] proposes a torque calculation model that incorporates the boundary conditions of the lubricant flow field and variation of oil film temperature. The torque transfer equation of a multi-plate clutch as related to viscosity was derived by combining the three-dimensional Navier-Stokes equation with the boundary conditions of the lubricant flow field. Taking the vehicle transfer case as an example, the equation for the variation in oil film radial temperature was obtained through the thermodynamic model of a flow field. By applying the parameters derived from the temperature equation to the PWA torque transfer model, the values under four working conditions were calculated. The results show that the relative errors between the corrected torque value and the test value under the four working conditions were less than 10%. Comparative analysis of simulation and experiment shows that PWA has advantages. In paper [2] the owing to such a significant performance of heat transfer, the steady slip flow and heat transfer of tangent hyperbolic fluid over a lubricating surface of the stretchable rotatory disk is investigated. The governing nonlinear partial differential equations (PDEs) have been converted into ordinary differential equations (ODEs), which are solved numerically using the Keller-box method. The upshots of pertinent parameters upon the dimensionless distributions of velocity and temperature are deliberated. The surface drag forces and heat transfer rates are computed, and the effects of governing parameters on them are examined. In [3] the lubrication of the piston skirt-cylinder interface involves multiple physical fields, and these physical fields are coupled. A new method is proposed in this study for modeling and analysis of the lubricated piston skirt-liner interface with multi-physics coupling. The results indicate that although a thinner skirt will affect the stability of piston motion and increase the slapping noise and wear, it will benefit hydrodynamic lubrication between skirt and cylinder and reduce friction power loss. In study [4] the influence of two thermal effects in a low-speed two-stroke diesel engine on piston ring lubrication performance are discussed in detail. For the TDCL, a steady-state heat-transfer model based on sequential fluid–solid coupling is used to solve for the heat-transfer coefficient and temperature on the coupling surface between the cylinder liner and the cooling water jacket. The boundary condition used to calculate the TDCL is the temperature distribution on the inner surface of the cylinder liner calculated empirically and verified experimentally. The results of the TDCL and the ASTM model are used in the lubrication model to analyze how the thermal effects influence the lubrication performance of the piston-ring–liner tribological pair. A fluid-structure coupling numerical model of an oil-jet lubricated cylindrical roller bearing (CRB) in a high-power gearbox is developed, in which the volume of fluid (VOF) method and slip mesh model are used. The heat sources of bearing are defined as the transient thermal boundary in [5]. The differences of temperatures between the rigid and flexible bearings are compared. The simulation results are validated by the presented experimental results. It indicates that the oil flow rate, speed, waviness amplitude and order, oil viscosity and nozzle number will significantly influence the bearing’s lubricating characteristics. The flexible bearing model with the transient thermal boundary is more accurate than those of other two models. To suppression the increment of bearing temperature, the waviness should be reduced. Advancements in mechanical expertise and rigorous need for gyratory components of machines expedite scientists towards essentiality of the eternal evolution of modified lubricants to corroborate the reliability, innocuous procedure and stability of sundry bearings. To enhance the performances at heftily ponderous load and high velocity, the high molecular polymers are utilized in mechanical bearings as lubricant. These lubricants [6] are non-Newtonian in characteristics and comply with different constitutive relationships. One of them is the power law lubricant which complies with Ostwald model and is broadly utilized for the engineering lubrication. The derivation of the slip-flow conditions at the interface of bulk Jeffrey fluid and the thin layer non-Newtonian lubricant is performed over a disk spiraling with simultaneous radial stretching and rotation. To obtain a homogeneous solution, the power-law index is suggested 1/3 and no-slip boundary condition is converted into an incipient slip boundary condition. A single dimensionless slip parameter is introduced to regulate the velocity slip. Flow equations are obtained, and the similarity conversion is performed to obtain ordinary differential equations. Numerical results are computed to visually perceive the effects of lubrication on the flow field by incorporating the interfacial slip conditions. The presence of the lubrication enhances the fluid velocity and plays major role in the reduction of the skin friction at disk surface. Thus, the creation of models of heat and mass transfer during boundary lubrication is a modern problem.

Main material. Models of the formation of transition temperatures during boundary lubrication

The gradient law of Fourier thermal conductivity can be used in the formation of boundary oil films:

$$q_t = -\lambda \Delta T = -\lambda g_t , \tag{1}$$

where q_t is the heat gradient;

$g_t = \mathrm{grad} T = \nabla T$ is the temperature gradient;

λ is the thermal conductivity, [11].

To summarize Fourier's law, the Maxwell-Cattaneo relationship:

$$q_t = -\lambda g_t - \tau_t \frac{\partial q_t}{\partial \tau_{\ t}} , \tag{2}$$

where τ_t is the time or period of relaxation of the temperature field.

It is established [5, 6] that the isotropic solid under the conditions of lubrication is affected by the hyperbolic equation of heat transfer:

$$\frac{\partial T}{\partial \tau} + \tau_t \frac{\partial^2 T}{\partial \tau^2} = \alpha \Delta T + \frac{\gamma_t}{\rho c_p} , \tag{3}$$

where ρ is the the density of the lubricating film;

c_p is the її molar heat capacity;

γ_t is the density of distribution of heat energy drains.

The solution of equation (3) with the necessary initial and boundary conditions corresponds to the propagation of the temperature field with a sharply delineated front, which propagates with velocity:

$$V_t = \sqrt{\frac{a}{\tau_t}} ,$$

де a is the coefficient of thermal conductivity.

$$a = \frac{\lambda}{\rho c_p} . \tag{4}$$

Therefore, equation (3) describes the process of heat propagation with a finite velocity V_t.

However, the process of heat distribution in the formation of lubricating films is not Markov, ie the magnitude of heat flux is determined by the entire "history" of heat transfer in some elementary volume.

With this approach, the equation of the relationship between heat flux and temperature gradient will take the form

:

$$g_t = -\int_0^{\Delta T} k(\theta) g(\tau - \theta) d\theta , \tag{5}$$

where $k(\theta)$ is the lubrication film temperature field relaxation function;

τ, θ is the coordinates of this field.

If $g(\tau - \theta) = g(\tau)$, then equation (5) passes into (1), and:

$$\lambda = \int_0^{\Delta T} k(\theta) d\theta . \tag{6}$$

Assuming that the relaxation $k(\theta)$ function attenuates exponentially, ie:

$$k(\theta) = \frac{\lambda}{\tau_t} \exp\left(-\frac{\theta}{\tau_t} \right) , \tag{7}$$

you can go from (5) to (2).

The heat flux q_t and internal energy e of the film, taking into account the relaxation functions, can be found as follows:

$$\begin{cases} q_t = -\int_0^{\Delta T} k(\theta) g'(\theta) d\theta; \\ e = e_0 + cT - \int_0^{\Delta T} l'(\theta) T'(\theta) d\theta, \end{cases} \tag{8}$$

Where value $g'(\theta)$ and $T'(\theta)$ determined by the ratios:

$$\begin{cases} \frac{d}{d\theta} g'(\theta) = g(\tau - \theta); \\ \frac{d}{d\theta} T'(\theta) = T(\tau - \theta) \end{cases}, \tag{9}$$

where c is the volumetric heat capacity of the oil;

$l(\theta)$ is the relaxation function of internal energy, [4, 5].

Find the time derivatives of heat flux and internal energy:

$$\begin{cases} \frac{\partial q_t}{\partial \tau} = -k(\theta) g - \int_0^{\Delta T} k'(\theta) g(\tau - \theta) d\theta; \\ \frac{\partial e}{\partial \tau} = c \frac{\partial T}{\partial \tau} + l(\theta) T + \int_0^{\Delta T} l'(\theta) T(\tau - \theta) d\theta. \end{cases} \tag{10}$$

Using the conservation equation for the internal energy in the form:

$$\frac{\partial e}{\partial \tau} = div q_t + \gamma_t, \tag{11}$$

we obtain the following integro-differential equation of heat transfer in the lubricating film, considering the lubricant an isotropic material:

$$c \frac{\partial^2 T}{\partial \tau^2} + l(\theta) \frac{\partial T}{\partial \tau} + \int_0^{\Delta T} l'(\theta) \frac{\partial T(\tau - \theta)}{\partial \tau} d\theta = k(\theta) \Delta T + \int_0^{\Delta T} k'(\theta) \Delta T(\tau - \theta) d\theta + \gamma_t . \tag{12}$$

The heat flux determined by relation (5) does not explicitly depend on the instantaneous value of the temperature gradient, but is determined by the entire "history" of heat transfer. However, in explicit form, equation (5) will have the form:

$$q_t = -k(\theta) g - \int_0^{\Delta T} k'(\theta) g(\tau - \theta) d\theta . \tag{12}$$

If we use the equation for the internal energy of the film in the form:

$$l = e_0 + l(\theta) T + \int_0^{\Delta T} l'(\theta) T(\tau - \theta) d\theta , \tag{14}$$

then equation (12) will take the form:

$$l(\theta) \frac{\partial T}{\partial \tau} + \int_0^{\Delta T} l'(\theta) \frac{\partial T(\tau - \theta)}{\partial \tau} d\theta = k(\theta) \Delta T + \int_0^{\Delta T} k'(\theta) \Delta T(\tau - \theta) d\theta + \gamma_t . \tag{15}$$

At $k(\theta) \to 0$ this equation will take the form:

$$l(\theta)\frac{\partial T}{\partial \tau} + \int_0^{\Delta T} l'(\theta)\frac{\partial T(\tau-\theta)}{\partial \tau}d\theta = \int_0^{\Delta T} k'(\theta)\Delta T(\tau-\theta)d\theta + \gamma_t \,. \tag{16}$$

Equation (16) can be represented in the form close to (12):

$$l(\theta)\frac{\partial^2 T}{\partial \tau^2} + l'(\theta)\frac{\partial T}{\partial \tau} + \int_0^{\Delta T} l''(\theta)\frac{\partial T(\tau-\theta)}{\partial \tau}d\theta = k'(\theta)\Delta T + \int_0^{\Delta T} k''(\theta)\Delta T(\tau-\theta)d\theta + \gamma_t \,. \tag{17}$$

Equation (17) can be considered a mathematical model of the phenomenon of heat transfer by the lubricating film up to the formation of its limit state at temperature T_{cr1}, determined according to [7, 8] ($T_{cr1} = \Delta T$).

The equation of motion of the lubricating film on the surface of the lubricated body is obtained from the equation of motion for the Newtonian continuous medium, [11]:

$$\frac{\partial v}{\partial \tau} + (\nabla v)v = g - \frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\Delta v + \frac{1}{\rho}\left(\xi + \frac{1}{3}\mu\right)\operatorname{grad}\operatorname{div} v\,, \tag{18}$$

where p is the contact stresses in the lubrication zone;
v is the the average speed of movement of the lubricating film on the body surface;
μ is the chemical potential of the surface, [11];
ξ is the chemical potential reduction factor.

In the case of uncompressed solid medium in the oil, when the value $\operatorname{grad}\operatorname{div} V$ rather small, equation (18) turns into the Navier-Stokes equation:

$$\frac{\partial v}{\partial \tau} + (\nabla v)v = g - \frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\Delta v\,. \tag{19}$$

In mathematical modeling of the processes of physicochemical transformations of the lubricating film until the values are reached T_{ch} i T_{cr2} we will analyze the phenomenon of mass transfer in the film itself. To do this, we use Fick's diffusion law [10, 11, 12]:

$$q = -De\Delta\rho\,, \tag{20}$$

where De is the effective diffusion coefficient;
q is the irreversible flow of mass transfer.

Lubrication filtration transfer in the first approximation obeys Darcy's law, [5]:

$$v = -\frac{k}{\mu}\nabla p\,, \tag{21}$$

where k is the the permeability coefficient of the porous surface structure;
μ is the dynamic viscosity coefficient of the lubricating film;
v is the average mass transfer rate of wear products.

From (20) and (21) after the corresponding transformations we obtain the mass transfer equation with a lubricating film:

$$\varepsilon\frac{\partial \rho}{\partial \tau} = De\Delta\rho + \frac{k}{\mu}\nabla\left(\rho\nabla\rho\right) - \frac{k}{\mu}g\nabla\rho^2 + \nu\,, \tag{22}$$

where ε is the porosity or relative fraction of voids of the lubricant:

$$\varepsilon = \lim_{V \to 0} \frac{V - V_s}{V}, \tag{23}$$

where V is the the volume of the element of the porous structure;
V_s is the the total volume of the rigid frame of the same element;
ν is the density of surface bodies, $\nu = \rho g$;
g is the acceleration of gravity.

The transfer potential in the process of mass transfer is the chemical potential μ_i transferable component. The chemical potential is related to the concentration of the component ρ_i , with:

$$\nabla\mu_i = \left(\frac{\partial\mu_i}{\partial\rho_i}\right)_{p,T}, \nabla\rho_i + \left(\frac{\partial\mu_i}{\partial T}\right)_{\rho_i,T} \nabla T + \left(\frac{\partial\mu_i}{\partial\rho}\right)_{\rho_i,p} \nabla p. \tag{24}$$

For mass transfer at constant pressure, contact stresses and in isothermal conditions:

$$\nabla\mu_i = \left(\frac{\partial\mu_i}{\partial\rho_i}\right)_{p,T} \nabla\rho_i, \tag{25}$$

If the deviations of our system from the state of thermodynamic equilibrium are not so large, then the mass transfer flow can be described by a linear gradient law:

$$q_{mi} = -K\nabla\mu_i = -K\left(\frac{\partial\mu_i}{\partial\rho_i}\right)_{p,T} \nabla\rho_i = -D_i\nabla\rho_i. \tag{26}$$

Ziegler, [11], showed that the principle of thermodynamics of irreversible processes in combination with the principle of the smallest irreversible forces gives the maximum speed of dissipation. The speed of dissipation (dissipative function of the system) is determined by the ratio:

$$D(q_m) = \frac{dW}{d\tau} = g(q_m) q_m, \tag{26*}$$

where $g(q_m)$ is the generalized thermodynamic force that is proportional to the gradient of chemical potential.

Dissipative function can be introduced without the aid of irreversible force:

$$D(q_m) = T\left(\frac{dS}{d\tau}\right)_{ir} \geq 0, \tag{27}$$

де $\left(\frac{dS}{d\tau}\right)_{ir}$ is the entropy flow due to the irreversible process of mass transfer.

The principle of maximum speed of dissipation requires ensuring the maximum $Д(q_m) = g(q_m) q_m$ due to the appropriate choice of transfer flow in the presence of restrictions on the species $F(q_m) = 0$. We get the solution from the relation:

$$\frac{\partial}{\partial q_m}\left[g(q_m) q_m - \lambda F(q_m)\right] = 0. \tag{28}$$

Condition $F(q_m) = 0$, де $F(q_m) = Д(q_m) - \mathrm{g}(q_m) q_m$ is a condition for the stability of the process of transition of the system to a state of thermodynamic equilibrium. From relation (28) it follows that:

$$g\left(q_m\right)=\frac{\lambda}{1+\lambda}\frac{\partial D}{\partial q_m}. \tag{29}$$

Determining the Lagrange multiplier λ, can be obtained:

$$g\left(q_m\right)=\left(\frac{\partial D}{\partial q_m}\right)^{-1} D\frac{\partial D}{\partial q_m}. \tag{30}$$

Dissipative functions of homogeneous processes satisfy the functional equation:

$$\frac{\partial D}{\partial q_m}q_m = f\left(D\right). \tag{31}$$

The condition of stability has the form:

$$f\left(D\right)>D \text{ при } D>0. \tag{32}$$

Possible dissipative functions for the mass transfer process must be solutions of equation (31) with condition (32).

A special case (31) is equation:

$$\frac{\partial D}{\partial q_m}q_m = \alpha D. \tag{33}$$

The stability condition (32) in this case will be:

$$\alpha>1. \tag{34}$$

Equation (30) can be converted by (31) into a more convenient relationship:

$$g\left(q_m\right)=\frac{D}{f\left(D\right)}\cdot\frac{\partial D}{\partial q_m}. \tag{35}$$

The solution of functional equation (33) will be the dissipative function of the form:

$$D=\text{const}\cdot q_m^{\alpha},\quad \alpha>1. \tag{36}$$

Using equations (35) and (36) we can obtain the following relationships between the gradient of chemical potential and mass transfer flow:

$$g=\text{const}\cdot q_m^{\alpha-1},\ \alpha>1. \tag{37}$$

Non-Markov process confirms that the magnitude of the mass transfer flow is determined by the entire "history" of mass transfer:

$$q_m=-\int_0^{\Delta T} M\left(\theta\right)g^n\left(\tau-\theta\right)d\theta,\ \ n\le 1. \tag{38}$$

If we explicitly determine the dependence of the magnitude of the transfer flux on the instantaneous value of the gradient of the chemical potential, the relationship (38) will take the form:

$$q_m=-M\left(\theta\right)g^n-\int_0^{\Delta T} M'g^n\left(\tau-\theta\right)d\theta,\ \ n\le 1. \tag{39}$$

Assumptions about the exponential attenuation of the relaxation function, ie:

$$M(\theta)=\frac{m}{\tau_m}\exp\left(-\frac{\theta}{\tau_m}\right), \tag{40}$$

converts (39) into a ratio of the form:

$$q_m=-[M(\theta)+m]g^n-\tau_m\frac{\partial q_m}{\partial \tau}. \tag{41}$$

In formulas (38) - (41) g is thegradient of the chemical potencial.

Therefore, formulas (22) - (41) represent a mathematical model of the actual mass transfer of the lubricating film in the formation of its limit state. From formulas (22) - (41) follows the value T_{ch} and T_{cr2}, [8, 9].

Conclusion

The nonlinear models of heat- and mass transfer for determination transition of temperatures by forming boundary lubricant film according to the conception of structure-thermodynamic approach of description the process of boundary lubricating is developed.

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Диха О.В., Старий А.Л. Моделі тепломасообміну при граничному змащуванні для визначення перехідних температур

В даний час все більше розвиваються кінетичні та термодинамічні методи оцінки змащувальної дії масел. При граничному терті зменшення тертя і зносу поверхонь відбувається за рахунок здатності мастила утворювати на поверхні шари адсорбційного або хімічного походження. Для математичного опису процесів у підсистемах та переходу між ними необхідно використовувати аналітичні моделі переходів температури та зносу в режимі граничного змащування. За базову розрахункову залежність прийнято рівняння Фур'є теплопровідності. Передбачається, що процес поширення тепла за умов утворення мастильних плівок не є марковським, тобто величина теплового потоку визначається всією «історією» теплообміну в певному елементарному об’ємі. Рівняння руху мастильної плівки по поверхні тіла, що змащується, виходить з рівняння руху для ньютонівського безперервного середовища. В результаті отримано нелінійні моделі тепломасообміну для визначення температур переходу при формуванні граничних мастильних плівок у концепції структурно-термодинамічних підходів до опису процесів граничного змащування поверхонь.

Ключьові слова: граничне мащення, перехідні температури, математична модель