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Mathematical model of running-in of tribosystems under conditions of boundary lubrication. Part 1. Development of a mathematical model

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Abstract

The paper further developed the methodological approach in obtaining mathematical models that describe the running-in of tribosystems under boundary lubrication conditions.

The structural and parametric identification of the tribosystem as an object of simulation of running-in under conditions of extreme lubrication was carried out. It has been established that the processes of running-in of tribosystems are described by a second-order differential equation and, unlike the known ones, take into account the limit of loss of stability (robustness reserve) of tribosystems. It is shown that the nature of tribosystems running-in conditions of extreme lubrication depends on the gain coefficients and time constants, which are included in the right-hand side of the differential equation.

It is shown that the processes of running-in of tribosystems depend on the type of the magnitude of the input influence on the tribosystem, the first and second derivatives. The input influence is represented as a product of coefficients and a time constant $K_0 \cdot K_2 \cdot T_3$. This allows us to state that the processes of the tribosystem running-in will effectively take place when the input action (load and sliding speed), will change in time and have fluctuations with positive and negative acceleration of these values from the set (program) value. This requirement corresponds to the running-in program "at the border of seizing".

The left part of the equation is the response of the tribosystem to the input signal. Tribosystem time constants T_2 and T_3 have the dimension of time and characterize the inertia of the processes occurring in the tribosystem during running-in. Increasing the time constants makes the process less sensitive to changes in the input signal, the warm-up process increases in time, and the tribosystem becomes insensitive to small changes in load and sliding speed. Conversely, the reduction of time constants makes the tribosystem sensitive to any external changes.

Keywords: tribosystem; running-in; mathematical model of running-in; differential equation; gain; time constant; boundary lubrication; quality factor of the tribosystem; robustness of the tribosystem; volumetric wear rate; coefficient of friction

Introduction

The running-in of tribosystems is the final technological stage in the production process of machines and the initial stage of operation of these machines. In the process of running-in, the tribosystems that make up the machine or unit form bearing surface layers, providing further maximum resource and minimum friction losses. An analysis of the publications of many researchers who are devoted to running in or running-in allows us to state that the completion of the running-in process is reduced not only to the formation of the optimal roughness of the mating friction surfaces. The running-in process includes physical and chemical phenomena, such as thermal, diffusion, deformation, which take place on actual contact spots in the presence of lubricating media and the environment. Therefore, reducing the running-in process, with a simultaneous decrease in wear for running-in and friction losses, will significantly increase the resource of machines and mechanisms, which will provide an economic effect during operation.

An analysis of models of stationary processes and running-in processes in tribosystems shows that there is a large error in modeling the wear rate, up to 18,8%, and the friction coefficient, up to 16,0%. Such a scatter of data in measurements can be explained by the presence of an oscillatory process of wear rate and friction

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coefficient during the running-in of the tribosystem, as well as by the ambiguity of the choice of input parameters for modeling. Difficulties that arise in the development of such models are associated with the choice of parameters that affect the process under study. For example, the design of the tribosystem, the lubricating medium, the materials from which the triboelements are made, the roughness of the friction surfaces, the load-speed range of operation, etc. The listed parameters are random functions, which makes it difficult to build mathematical models.

Literature review

An overview work, which is devoted to the processes of running-in, can be considered the work [1]. In this paper, a system analysis and comprehensive studies of the running-in processes are carried out, on the basis of which the definition of the running-in process is formulated. According to the authors, this is a transient process, including a complex interaction between friction surfaces, lubrication, roughness, plastic deformation and wear. The running-in process involves changing key tribological parameters such as surface roughness, surface topography, coefficient of friction and wear rate until a steady state prevails. The paper provides a review of the literature on the running-in processes.

The authors of the works [2-4] analysis of various types of running-in was carried out, where the change in the roughness of the friction surface during running-in is studied. For example, at work [2] developed a model for changing the roughness of friction surfaces as a function of time. The proposed model is non-linear, the optimal values of the model parameters were estimated using the Gauss-Newton algorithm. Experimental results taken from the literature for steel and alloy samples Cu-Zn were used to test the model and confirmed its information content. In works [3, 4], to reduce the running-in time, the initial roughness of the friction surface has been optimized. The authors established a correlation between the initial roughness and deformation processes of surface layers, showed ways to optimize the running-in of tribosystems.

In works [5, 6] the processes of mechanical treatment of friction surfaces with the formation of optimal roughness and its influence on the mechanisms of plastic deformation of surface layers during running-in were studied. On the basis of experimental data, a model of wear during running-in has been developed and the fact of grain refinement of materials of surface layers has been established.

The study of friction surfaces and the formation of films on them during the running-in process was carried out by the authors of the works [7-9]. The authors conclude that the change in friction and wear parameters that occur during running-in are not only the result of changes in surface roughness, but also the microstructure of the surface layers and the formation of a third body. The running-in process is described by piecewise models, which allow modeling not only roughness, but also the formation of a third body as a function of time. Authors of the work [8] claim that the running-in process can be controlled by changing the initial roughness and lubricating medium. For example, at work [9] the evolution of tribofilms on the friction surface during running-in is shown.

Works [10, 11] devoted to the development of models for the running-in of tribosystems. For example, at work [10] the tribosystem is presented as a running-in attractor, which is a stable and time-ordered structure that is formed in the running-in process. The authors carried out a dispersion analysis of the characteristic parameters of the running-in attractors to identify primary and secondary factors that affect the running-in process. According to the authors, the developed models make it possible to predict the running-in process of tribosystems. In work [11] a two-scale model of the formation of the topography of the friction surface during running-in is presented. This model makes it possible to determine the stresses on the actual contact spots and optimize the running-in processes.

The authors of the work [12] the analysis of the influence of the spectral load during the running-in of tribosystems was carried out. The results of experimental studies are discussed and a conclusion is made about the prospects of changing the load, along a given spectrum, during running-in. This approach improves the efficiency of running-in processes.

A similar approach is presented in the work [13]. The authors developed and substantiated the structure of the tribosystems running-in program, which consists of two modes. In the first mode, the maximum load is set, below the “sticking” load at the minimum sliding speed. This mode allows, due to the intense deformation of microprotrusions, to form an equilibrium roughness of friction surfaces and change the structure of thin surface layers. The first mode is called the adaptation of the tribosystem to external conditions. The second mode sets the minimum load and maximum sliding speed. This mode allows to reduce the time of restructuring of the material structure of the surface layers and to complete the formation of secondary structures and oxide films. The second mode is called the trainability and trainability of the tribosystem. The paper presents the transient characteristics of the running-in of tribosystems, which make it possible to establish the relationship between the design of the tribosystem, rational loading modes, running-in time and wear for running-in. The practical significance of the work is to minimize the run-in time and wear during the run-in period.

Summing up the analysis of works devoted to the running-in processes, we can conclude that the novelty of this study is the development of a mathematical model of the running-in process of tribosystems, which will allow modeling the change in wear rate and friction coefficient over time. The study of such processes will make it possible to substantiate the running-in regimes and reduce running-in wear and running-in time, as well as develop a program for effective running-in of various designs of tribosystems. Such models should determine the boundary of the stable operation of the tribosystem, i.e. the boundary of the tribosystem exit to the scoring or the boundary, when the accelerated wear of the materials of the triboelements begins [14]. Accounting for such regimes will improve the efficiency of modeling the running-in processes of tribosystems.

Purpose

The purpose of this study is to develop a mathematical model of running-in processes in tribosystems in the form of a differential equation and its solutions, which will allow modeling the wear rate and friction coefficient over time, taking into account the robustness range.

Methods

The structural identification of the mathematical model of tribosystems running in conditions of extreme lubrication will be performed according to the following structural and dynamic scheme, which is shown in fig. 1.

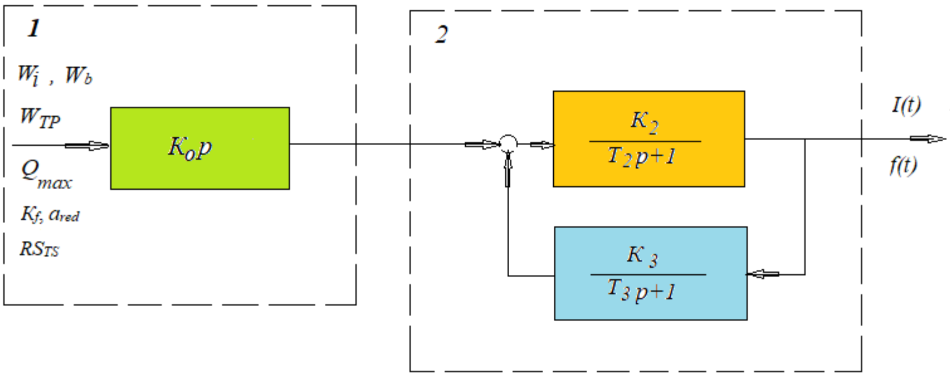

Fig. 1. Structural and dynamic scheme of simulation of tribosystems running-in processes

The structural dynamic scheme is built on the principle of two blocks connected in series.

The first block simulates the change of the following values:

- input impact on the tribosystem W_i, (the power supplied to the tribosystem) is determined by the formula given in the work [15];
- the maximum value of the input impact, when there is accelerated wear of tribosystem materials, or burr of friction surfaces, W_b, is determined by the formula given in the work [14];
- speed of dissipation in the tribosystem W_{TR}, is determined by the formula given in the work [15];
- the maximum value of the quality factor (Q-factor) of the tribosystem Q_{max} during the running-in time, is determined by the formula given in the work [16];
- the design parameters of the tribosystem are taken into account by the form factor K_f, is determined by the formula given in the work [16];
- given value of the coefficient of thermal conductivity of triboelement materials a_{red}, is determined by the formula given in the work [16];
- rheological properties of the structure of composite materials in the tribosystem RS_{TS}, is determined by the formula given in the work [16].

The second block of the structural-dynamic scheme, fig. 1, simulates the reaction of the tribosystem to a change in the input external influence, with a subsequent change and stabilization of the volumetric wear rate *I(t)* and coefficient of friction *f(t)* after completion of the running-in. Such processes are accompanied by a change in the roughness and structure of the surface layers of triboelement materials.

The transfer functions of the second modeling block, fig. 1, are similar to the transfer functions of the second block of the structural-dynamic scheme, which is presented in the work [15]. Transfer functions are inertial links and characterize the sensitivity of the tribosystem to input external influences and the ability of the tribosystem to rearrange the surface layers of the materials from which the triboelements are made during running-in. Such a reconstruction is associated with a change in the load, sliding speed, and Q-factor of the tribosystem [16]. Such processes are a function of time.

Applying the methods of the theory of identification of dynamic objects, it is possible, for the structural-dynamic scheme in fig. 1, to obtain an equivalent transfer function for simulating the running-in of the tribosystem:

$$G_{eq} = \frac{K_0 p \cdot K_2 (T_3 p + 1)}{(T_2 T_3) p^2 + (T_2 + T_3) p + K_2 K_3 + 1} .. \quad (1)$$

We will write the corresponding equation of the tribosystem running-in dynamics in the following form:

$$(T_2T_3)p^2 + (T_2 + T_3)p + K_2K_3 + 1 = (K_0K_2T_3)p^2 + K_0K_2p \quad (2)$$

The differential equation of the second order is written in operator form, where the symbol *p*, is the differentiation operator, *d/dt*.

The right-hand side of the differential equation (2), which characterizes the input effect on the tribosystem, contains the first and second derivatives. The input influence is represented as a product of coefficients and a time constant $K_0 \cdot K_2 \cdot T_3$. This allows us to state that the running-in processes of the tribosystem will effectively take place when the input action (load and sliding speed) will change in time and have fluctuations with positive and negative acceleration of these values from the set (program) value. This requirement corresponds to the running-in program "at the border of seizing".

The left part of the equation is the response of the tribosystem to the input signal. Tribosystem time constants T_2 and T_3 have the dimension of time and characterize the inertia of the processes occurring in the tribosystem during running-in.

Increasing time constants T_2, and T_3, makes the process less susceptible to changes in the input influence, the running-in process increases in time, and the tribosystem becomes insensitive to minor changes in load and sliding speed. Conversely, the reduction of time constants makes the tribosystem sensitive to any external changes.

The procedure of parametric identification or finding expressions for calculating gain coefficients and time constants that characterize the dynamics of the tribosystem run-in process is an experimental material that allows you to select the most significant factors that affect the running-in process.

Gain factor K_0, which is included in the first block of the structural-dynamic diagram, fig. 1 and in the differential equation, formula (2) takes into account the degree of influence of the input signal (load, sliding speed, tribological characteristics of the lubricating medium) on the value of the output signal (Q-factor of the tribosystem). Based on this physical concept and using the dimensional methods of similarity theory and modeling, we will get an expression:

$$K_0 = \frac{W_i}{W_b} \cdot \quad (3)$$

where W_i - the input effect, or the power supplied to the tribosystem, is calculated as the product of the load and the sliding speed, the formula for the calculation is given in works [14, 15];

W_b - input impact, or power, when loss of stability, burr or accelerated wear occurs, the formula for calculation is given in the work [14].

As follows from the expression (3) of the ratio of the active input influence W_i, selected for the tribosystem running-in mode, to the maximum value W_b, when there is a loss of stability, burr or accelerated wear, characterizes the maximum value W_i, which can be used when running in tribosystems. Relation W_i/W_b should not exceed units. Size W_b is determined by modeling according to the method given in the work [14].

The physical meaning of the coefficient K_2 – it is the sensitivity of the tribosystem to changes in external influences (load, sliding speed, Q-factor of the tribosystem). The value of the coefficient K_2 is calculated according to the formula given in the work [14] and has a similar physical meaning.

Coefficient K_3 – characterizes the ability of the tribosystem to self-organize when the values of the input parameters (load, sliding speed, Q-factor of the tribosystem) change. The value of the coefficient K_3 is calculated according to the formula given in the work [14] and has a similar physical meaning.

Time constant T_2, which is included in the left part of the differential equation (2), characterizes the time during which the temperature gradient stabilizes by the volumes of the triboelements, taking into account the thermal conductivity of materials when the external conditions change, the dimension is a second. Value T_2 is calculated according to the formula given in the work [14] and has a similar physical meaning.

Time constant T_3, which is included in both the right and left parts of the differential equation (2), characterizes the time during which the tribosystem returns to a stable mode of operation after the cessation of the action of the disturbing force, or the time until the parameters stabilize in the new mode of operation. Value T_3 is calculated according to the formulas given in the work [14] and have a similar physical meaning.

Results

The solution to the above differential equation (2) when simulating the volume rate of wear is the following expression:

$$I(t) = I_{st}\left[1 + (K_0 \cdot K_2)^{\lambda}(t) \cdot e^{\left(-\frac{d_I}{0,3 \cdot T_I} \cdot t\right)} \cdot (\cos \upsilon_I t + A_I \sin \upsilon_I t)\right], \quad (4)$$

where I_{st} – the value of the wear rate of the tribosystem after running-in (stationary mode) is determined by the expression given in the work [18];

λ – exponent, which takes into account the change in the constant T_3, as a function of running-in time, a dimensionless quantity;

t – running-in time, which varies from zero to completion of the running-in process, dimension second.

The decrement of damping of oscillations during running-in is represented by the following formula:

$$d_I = \frac{(T_2 + T_{3(I)})}{2 \cdot T_I}. \tag{5}$$

The time constant of the tribosystem for simulating the volumetric rate of wear during running-in is represented by the following formula:

$$T_I = \sqrt{T_2 \cdot T_{3(I)}} . \tag{6}$$

The frequency of wear rate fluctuations υ_I during running-in:

$$\upsilon_I = \frac{\sqrt{1 - d_I^2}}{T_I} \tag{7}$$

The amount of deviation of the volume rate of wear from the current value during the oscillating process:

$$A_I = \frac{d_I}{\sqrt{1 - d_I^2}}. \tag{8}$$

The solution to the above differential equation (2) when modeling the friction coefficient is the following expression:

$$f(t) = f_{st}\left[1 - (K_0 \cdot K_2)^{\lambda}(t) \cdot e^{\left(-\frac{d_f}{0,3 \cdot T_f} \cdot t\right)} \cdot (\cos \upsilon_f t + A_f \sin \upsilon_f t)\right], \tag{9}$$

where f_{st} – the value of the friction coefficient of the tribosystem after running-in (stationary mode) is determined by the expression given in the work [18].

The decrement of damping of oscillations during running-in is represented by the following formula:

$$d_f = \frac{(T_2 + T_{3(f)})}{2 \cdot T_f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The time constant of the tribosystem for simulating the friction coefficient during running-in is represented by the following formula:

$$T_f = \sqrt{T_2 \cdot T_{3(f)}} . \tag{11}$$

Frequency of friction coefficient fluctuations υ_f during running-in:

$$\upsilon_f = \frac{\sqrt{1 - d_f^2}}{T_f}. \tag{12}$$

The amount of deviation of the friction coefficient from the current value during the oscillating process:

$$A_f = \frac{d_f}{\sqrt{1 - d_f^2}}. \tag{13}$$

When modeling the running-in processes of tribosystems, especially according to the program "at the boundary of seizing", it is necessary to take into account the limiting values of the load and sliding speed when accelerated wear or scuffing occurs. Such parameters are not constant for tribosystems, but depend on the design (shape factor), rheological properties of triboelement materials and their thermal diffusivity, roughness of friction surfaces and sliding speed. A change in the sliding speed leads to a change in the strain rate on the actual contact patches, which affects the quality factor of the tribosystem and its change during running-in, which is presented in the work [16].

On fig. 2 shows the dependences of the change in the magnitude of the input impact on the tribosystem when scoring or accelerated wear occurs – W_b, when changing the sliding speed and tribological properties of the lubricating medium. The dependencies are built according to the method for assessing the robustness of tribosystems, presented in the works [14, 19] and verified experimentally with an assessment of the reproducibility of the results and the adequacy of the simulation results to the experimental data.

The dependencies are built for the tribosystem «steel 40H + Br.AZh.9-4», K_f = 12,5 m^{-1}, *Ra* = 0,2 micron, *Sm* = 0,4 mm.

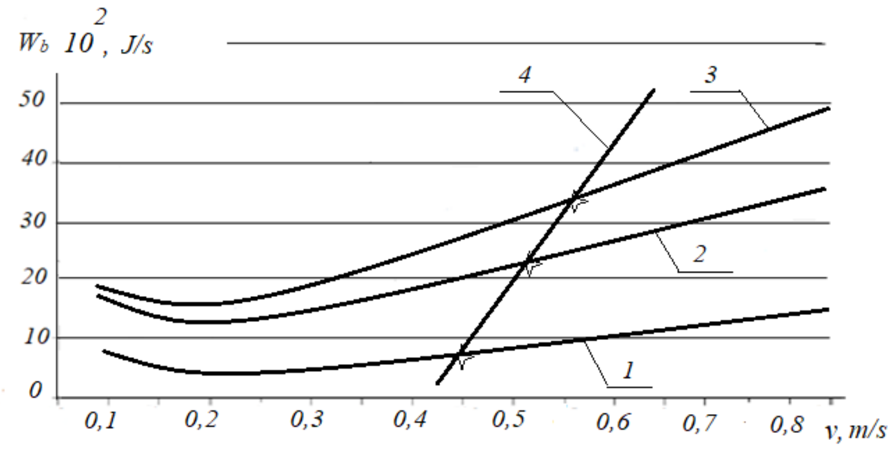

Fig. 2. Dependencies of the value of the input impact, when the loss of stability occurs (accelerated wear or burr), for different values of the sliding speed and tribological properties of the lubricating medium: 1 - MG-15B hydraulic oil; 2 - engine oil M-10G$_{2K}$; 3 - transmission oil TS$_P$-15K; 4 – the resulting curve for points that have the same loss of stability value

Analysis of the presented dependencies allows us to draw the following conclusions. The figure field can be divided by curve 4 into two parts. To the left of curve 4 - the loss of stability of tribosystems occurs due to the occurrence of accelerated wear. To the right of curve 4 - due to tearing of friction surfaces. Points on curves 1, 2, 3, marked with "stars" have the physical meaning of the points of transition of buckling from accelerated wear of friction surfaces to scuffing of surfaces.

As follows from the dependencies, curves 1, 2, 3 have a minimum, where the occurrence of accelerated wear occurs at the minimum values of the input action, which is supplied to the tribosystem. This minimum can be explained by the absence of protective structures on the friction surfaces, since activation energy is not enough for their formation. There is an adsorbed viscous lubricating film on the friction surface. With an increase in the power supplied to the tribosystem, the activation energy becomes sufficient to form, first, viscoelastic structures, and then, solid elastic structures (right side of line 4).

These dependencies formed the basis for obtaining the exponent λ, which is presented in formulas (4) and (9) and which can be expressed by the following relationship:

$$\lambda = \frac{T_3(v = v_{red})}{T_3(W_b = \min)} . \qquad (14)$$

where $T_3(v=v_{red})$ – the value of the time constant at the sliding speed, which corresponds to the tribosystem running-in mode, dimension s;

$T_3(W_b=min)$ – the value of the time constant at the sliding speed, which corresponds to the minimum value of the input influence, when the tribosystem loses stability, dimensionality s. This is the minimum on the curves, fig. 2.

When modeling the change in the volumetric wear rate during running-in, it is necessary to substitute the value in formula (14) $T_{3(I)}$, the formula for calculation is presented in the work [14]. When modeling the friction coefficient, in formula (14) it is necessary to substitute the value $T_{3(f)}$, the formula for calculation is presented in the work [14].

Exponent λ with the coefficients $K_0 K_2$ in the solutions of differential equations (4) and (9), takes into account the margin for stable operation of the tribosystem during running-in. Or, according to work [14] – tribosystem robustness margin. The method for determining the robustness of a tribosystem is described in the work [14].

Applying formulas (4) – (14), it is possible to simulate the processes of running-in of tribosystems over time when the following input factors are changed:

- load, N;

- sliding speed, m/s;
- geometric dimensions of the tribosystem, which are taken into account by the form factor K_a, 1/m;
- coefficients of thermal conductivity of materials of moving and fixed triboelements a_{red}, m^2/s;
- rheological properties of the structure of combined materials in the tribosystem RS_{TS}, 1/m;
- tribological properties of the lubricating medium E_u, J/m^3;
- roughness of friction surfaces *Ra* and *Sm*, m.

Conclusions

The structural and parametric identification of the tribosystem as an object of simulation of running-in under conditions of extreme lubrication was carried out. It has been established that the processes of running-in of tribosystems are described by a second-order differential equation and, unlike the known ones, take into account the limit of loss of stability (robustness reserve) of tribosystems. It is shown that the nature of tribosystems running-in conditions of extreme lubrication depends on the gain coefficients and time constants, which are included in the right-hand side of the differential equation.

It is shown that the processes of running-in of tribosystems depend on the type of the magnitude of the input influence on the tribosystem, the first and second derivatives. The input influence is represented as a product of coefficients and a time constant $K_0 \cdot K_2 \cdot T_3$. This allows us to state that the running-in processes of the tribosystem will effectively take place when the input action (load and sliding speed) will change in time and have fluctuations with positive and negative acceleration of these values from the set (program) value. This requirement corresponds to the running-in program "at the border of seizing".

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Войтов А.В. Математична модель припрацювання трибосистем в умовах граничного мащення. Частина 1. Розробка математичної моделі

У роботі отримав розвиток методичний підхід в отриманні математичних моделей, які описують припрацювання трибосистем в умовах граничного мащення.

Виконано структурну та параметричну ідентифікацію трибосистеми, як об'єкта моделювання припрацювання в умовах граничного мащення. Встановлено, що процеси припрацювання трибосистем описується диференційним рівнянням другого порядку та на відміну від відомих враховує межу втрати стійкості (запас робастності) трибосистем. Показано, що характер припрацювання трибосистем в умовах граничного мащення залежить від коефіцієнтів підсилення і постійних часу, які входять в праву частину диференційного рівняння.

Показано, що процеси припрацювання трибосистем залежать вид величини вхідного впливу на трибосистему, перша та друга похідні. Вхідний вплив представлено у вигляді добутку коефіцієнтів та постійної часу $K_0 \cdot K_2 \cdot T_3$. Це дозволяє стверджувати, що процеси припрацювання трибосистеми ефективно проходитимуть, коли вхідний вплив (навантаження і швидкість ковзання), змінюватимуться в часі і мати коливання з позитивним і негативним прискоренням цих величин від встановленого (програмного) значення. Такій вимогі відповідає програма припрацювання «на межі заїдання».

Ліва частина рівняння - це реакція трибосистеми на вхідний сигнал. Постійні часу трибосистеми T_2 та T_3 мають розмірність часу і характеризують інерційність процесів, що протікають в трибосистемі, під час припрацювання. Збільшення постійних часу робить процес менш сприйнятливим до зміни вхідного сигналу, процес припрацювання збільшується в часі, а трибосистема стає нечутливою до незначних змін навантаження та швидкості ковзання. І навпаки, зменшення постійних часу, робить трибосистему чутливою до будь яких зовнішніх змін.

Ключові слова: трибосистема; припрацювання; математична модель припрацювання; диференційне рівняння; коефіцієнт підсилення; постійна часу; граничне мащення; добротність трибосистеми; робастність трибосистеми; швидкість об'ємного зношування; коефіцієнт тертя